Shear Strength of Soils

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Outline

- Shear Failure
- Soil Strength
  - Mohr-Coulomb Failure Criterion
- Laboratory Shear Strength Test
  - Direct shear
  - Triaxial
- Stress Path
- Pore Pressure Parameters
**Shear failure**

Soils generally fail in **shear**

At failure, shear stress along the failure surface reaches the shear strength.

Shear failure

The soil grains slide over each other along the failure surface. No crushing of individual grains.
Shear failure

At failure, shear stress along the failure surface ($\tau$) reaches the shear strength ($\tau_f$).

Soil (Shear) Strength

- Soils are essentially frictional materials
  - the strength depends on the applied stress
- Strength is controlled by effective stresses
  - water pressures are required
- Soil strength depends on drainage
  - different strengths will be measured for a given soil
    a) deforms at constant volume (undrained) and
    b) deforms without developing excess pore pressures (drained)
Mohr-Coulomb Failure Criterion

\[ \tau_f = c' + \sigma' \tan \phi' \]

\( \tau_f \) is the maximum shear stress the soil can take without failure, under normal stress of \( \sigma' \).

Shear strength consists of two components: cohesive and frictional.
c’ and φ’ are measures of shear strength.

Higher the values, higher the shear strength.

The parameters c’, φ’ depend on

- **Soil composition**
- **Stress state of the soil (OCR)**

The Mohr-Coulomb criterion is an empirical criterion, and the failure locus is only locally linear. Extrapolation outside the range of normal stresses for which it has been determined is likely to be unreliable.

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**Mohr Circles & Failure Envelope**

Initially, Mohr circle is a point

The soil element does not fail if the Mohr circle is contained within the envelope.
Mohr Circles & Failure Envelope

As loading progresses, Mohr circle becomes larger...

.. and finally failure occurs when Mohr circle touches the envelope

Orientation of Failure Plane

Failure plane oriented at $45 + \phi'/2$ to horizontal ($\sigma'_1$)
Mohr circles in terms of $\sigma$ & $\sigma'$

Mohr-Coulomb

Envelopes in terms of $\sigma$ & $\sigma'$

Identical specimens initially subjected to different isotropic stresses ($\sigma_c$) and then loaded axially to failure.

At failure,

$\sigma_3 = \sigma_c$; $\sigma_1 = \sigma_c + \Delta \sigma_f$

$\sigma_3' = \sigma_3 - u_f$; $\sigma_1' = \sigma_1 - u_f$

Mohr-Coulomb
Effective stress failure criterion

If the soil is at failure the effective stress failure criterion will always be satisfied.

\[ \tau = c' + \sigma' \tan \phi \]

\(c'\) and \(\phi'\) are known as the effective (or drained) strength parameters.

Soil behaviour is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters. But they are not necessarily soil constants.

Total stress failure criterion

If the soil is taken to failure at constant volume (undrained) then the failure criterion can be written in terms of total stress as

\[ \tau = c_u + \sigma \tan \phi_u \]

\(c_u\) and \(\phi_u\) are known as the undrained strength parameters.

These parameters are not soil constants, they depend strongly on the moisture content of the soil.

The undrained strength is only relevant in practice to clayey soils that in the short term remain undrained. Note that as the pore pressures are unknown for undrained loading the effective stress failure criterion cannot be used.
Laboratory Tests for Shear Strength Parameters

- Direct shear test
- Triaxial test
- Direct simple shear test
- Plane strain triaxial test
- Torsional ring shear test

Direct Shear Test

- Top platen
- Normal load
- Motor drive
- Soil
- Load cell to measure Shear Force
- Porous plates
- Rollers

Measure
- Relative horizontal displacement, dx
- Vertical displacement of top platen, dy
Direct Shear

Sand

- Graph showing the relationship between shear stress and shear displacement for sand, with labels for peak shear strength and ultimate shear strength.

Clay

- Graph showing the relationship between shear stress and horizontal deformation for clay, with labels for peak shear strength and residual shear strength.

- Expressions for shear stress at failure:
  - Overconsolidated clay: \( \tau_f = c' + \sigma' \tan \phi' \) (\( c' \neq 0 \))
  - Normally consolidated clay: \( \tau_f = \sigma' \tan \phi' \) (\( c' = 0 \))

- Graph showing the relationship between effective normal stress and shear stress for clay.
Pros:
- Simplest and most economical for sandy soil
- Applicable for soil/structure interface

Cons:
- Soil not allowed to fail along the weakest plane.
- Shear stress distribution is not uniform.

Triaxial Test Apparatus

- Soil sample at failure
- Perspex cell
- Piston (to apply deviatoric stress)
- O-ring
- Impervious membrane
- Porous stone
- Water
- Cell pressure
- Back pressure
- Pedestal
- Pore pressure or volume change
Types of Triaxial Tests

Depending on whether drainage is allowed or not during initial isotropic cell pressure application, and shearing, there are three special types of triaxial tests that have practical significances. They are:

- Consolidated Drained (CD) test
- Consolidated Undrained (CU) test
- Unconsolidated Undrained (UU) test
Granular soils have no cohesion. \( c = 0 \) & \( c' = 0 \)

For normally consolidated clays, \( c' = 0 \) & \( c = 0 \).

For unconsolidated undrained test, in terms of total stresses, \( \phi_u = 0 \).

\[ \text{CD, CU and UU Triaxial Tests} \]

**Consolidated Drained (CD) Test**

- No excess pore pressure throughout the test
- Very slow shearing to avoid build-up of pore pressure
- Gives \( c' \) and \( \phi' \)

Use \( c' \) and \( \phi' \) for analysing fully drained situations (e.g., long term stability, very slow loading)
Loose sand / NC clay

Dense sand / OC Clay
CD, CU and UU Triaxial Tests

Consolidated Undrained (CU) Test

- Pore pressure develops during shear
- Gives $c'$ and $\phi'$
- Faster than CD (∴ preferred way to find $c'$ and $\phi'$)
Triaxial

Loose sand / NC clay

Dense sand / OC Clay

Effective stress failure envelope
\( \tau_f = \sigma' \tan \phi' \)

Total stress failure envelope
\( \tau_f = \sigma \tan \phi \)

Normal stress
Shear stress
CD, CU and UU Triaxial Tests

Unconsolidated Undrained (UU) Test

- Pore pressure develops during shear
  - Not measured: $\sigma'$ unknown
  - $= 0$; i.e., failure envelope is horizontal
- Analyze in terms of $\sigma$ $\Rightarrow$ gives $c_u$ and $\phi_u$
- Very quick test

Use $c_u$ and $\phi_u$ for analysing undrained situations (e.g., short term stability, quick loading)

UU test on saturated clay

![Graph showing Mohr's circles and failure envelope for UU test on saturated clay.](image)
The $\phi=0$ concept

Unconfined compression test on saturated clay
σ'₁ and σ'₃ at Failure

\[ \sin \phi' = \frac{\sigma'₁ - \sigma'₃}{c' \cot \phi' + \frac{\sigma'₁ + \sigma'₃}{2}} \]

\[ \tau_f = c' + \sigma' \tan \phi' \]

\[ \sigma'_1 = \delta'_3 \tan (45 + \phi') + 2c' \tan (45 + \phi') \]

\[ \sigma'_3 = \delta'_1 + 2c' \tan (45 - \phi') - 2c' \tan (45 - \phi') \]

Stress Point

\[ t = \frac{\sigma_v - \sigma_h}{2} \]

\[ S = \frac{\sigma_v + \sigma_h}{2} \]
Stress Path

During loading...

Stress path is the locus of stress points

Stress path is a convenient way to keep track of the progress in loading with respect to failure envelope.

Failure Envelopes

During loading (shearing)....
A simple way to estimate the pore pressure change in undrained loading, in terms of total stress changes ~ after Skempton (1954)

\[ \Delta u = B [\Delta \sigma_3 + A (\Delta \sigma_1 - \Delta \sigma_3) ] \]

Skempton’s pore pressure parameters A and B

For saturated soils, B \approx 1.

For normally consolidated clays \( A_f \approx 1 \).

For heavily overconsolidated clays \( A_f \) is negative.