Compressibility of Soil

Outline

- Introduction
- Immediate Settlement
- Primary Consolidation Settlement
  - Consolidation Settlement
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- Secondary Consolidation Settlement

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Soil Mechanics –
Compressibility of Soil
Introduction

Types of Settlements

- Immediate (elastic) settlement ($S_e$)
- Time-dependent
  - Primary consolidation ($S_c$) $\leftarrow$ transient pore pressure
  - Secondary consolidation ($S_s$) $\leftarrow$ creep
- Total settlement $S_T = S_c + S_s + S_e$
- For soft soils (i.e., clays): $S_c >> S_e > S_s$ (Usually)

(a) Leaning Tower, Pisa

(b) Palacio de las Bellas Artes, Mexico City
(The Palace of Fine Arts)
Contact Pressure and Settlement Profile

Immediate Settlement

Clay

Sand

Theory of Elasticity

\[ S_e = \Delta \sigma B \frac{1 - \mu_s^2}{E_s} I_p \]

Influence Factor (shape, position, foundation rigidity)
Improved Relationship for $S_e$

Equivalent diameter

$$S_e = \Delta \sigma B_e \frac{1 - \mu_s^2}{E_0} I_G I_F I_E$$

Foundation embedment

Foundation rigidity

Immediate Settlement

Immediate Settlement
Immediate Settlement

\[ K_F = \left( \frac{E_f}{E_0 + \frac{B_s}{2} k} \right) \left( \frac{2t}{B_s} \right)^3 \]

= Flexibility factor

\[ f \]
Fundamentals of Consolidation

\[ \Delta u = \Delta \sigma \]
\[ \Delta \sigma' = 0 \]

\[ \Delta u = 0 \]
\[ \Delta \sigma' = \Delta \sigma \]

Consolidation Settlement

Consolidation Settlement

Sand

Ground water table

Clay

Sand

Depth

\[ \Delta \sigma \]

(a)
What is Consolidation?

When a saturated clay is loaded externally, the water is squeezed out of the clay over a long time (due to low permeability of the clay).
What is Consolidation?

This leads to settlements occurring over a long time, which could be several years.

In granular soils...

Granular soils are freely drained, and thus the settlement is instantaneous.
During consolidation...

Due to a surcharge $q$ applied at the GL, the stresses and pore pressures are increased at $A$.

..and, they vary with time.

$\Delta \sigma$ remains the same ($=q$) during consolidation. $\Delta u$ decreases (due to drainage) while $\Delta \sigma'$ increases, transferring the load from water to the soil.
One Dimensional Consolidation

~ drainage and deformations are vertical (none laterally)
~ a simplification for solving consolidation problems

ΔH - Δe Relation

average vertical strain = \[
\frac{\Delta H}{H_o}
\]

Time = 0⁺

Time = ∞
Consider an element where $V_s = 1$ initially.

\[
\Delta e_e = \Delta e
\]

Time = 0\(^+\) Time = \(\infty\)

\[\therefore \text{average vertical strain} = \frac{\Delta e}{1 + e_o}\]

Equating the two expressions for average vertical strain,

\[
\frac{\Delta H}{H_o} = \frac{\Delta e}{1 + e_o}
\]

- consolidation settlement
- change in void ratio
- initial thickness of clay layer
- initial void ratio
Consolidation Settlement

Coefficient of compressibility
~ denoted by $a_v = \Delta e / \Delta \sigma$

Coefficient of volume compressibility
~ denoted by $m_v$
~ is the volumetric strain in a clay element per unit increase in stress

\[
m_v = \frac{\Delta V}{\Delta \sigma} \quad \text{kPa}^{-1} \text{ or MPa}^{-1}
\]

Consolidation Settlement

Consolidation Test
~ simulation of 1-D field consolidation in lab.

field

GL

lab

undisturbed soil specimen
Dia = 50-75 mm
Height = 20-30 mm

porous stone
metal ring (oedometer)
Consolidation Test

loading in increments
allowing full consolidation before next increment

\[ \Delta e_1 = \frac{\Delta H_1}{H_o} (1 + e_o) \]

\[ \Delta e_2 = \]

unloading
Consolidation Settlement

**e – log σ_v’ plot**

- from the above data

- loading
  σ_v’ increases &
  e decreases

- unloading
  σ_v’ decreases &
  e increases (swelling)

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Consolidation Settlement

**Compression and recompression indices**

- **Cc** ~ compression index

- **Cr** ~ recompression index
  (or swelling index)
Preconsolidation pressure is the maximum vertical effective stress the soil element has ever been subjected to.
Overconsolidation ratio (OCR)

\[ OCR = \frac{\sigma_p'}{\sigma_{vo}'} \]

Field

\( \sigma_{vo}' \)

\( \sigma_{vo} \)

\( e_0 \)

\( \sigma_p' \)

\( \log \sigma_v' \)

Consolidation Settlement

\( \delta \)-logt or e-logt plot for each loading step

Stage I: Initial compression

Stage II: Primary consolidation

Stage III: Secondary consolidation

\[ S_R = \frac{e_0 H_0}{1 + e_0} - \frac{e_1 H_0}{1 + e_1} = \frac{\Delta e H_0}{1 + e_0} \]

Primary settlement

Settlement, \( S \)
Consolidation Settlement

e-logσ' plot for all loading steps

Consolidation Settlement

Determining $C_c$, $C_r$, Preconsolidation Pressure

Casagrande's Method
Effect of Disturbance

Normally consolidated Clay of low to medium plasticity

Consolidation Settlement

Effect of Disturbance

Over-consolidated Clay of low to medium plasticity

Empirical equations

\[ C_c = 0.009 \times (LL - 10) \]

\[ C_s = \frac{1}{3} - \frac{1}{10} C_c \]
Calculating Consolidation Settlement

Settlement computations

Two different ways to estimate the consolidation settlement:

(a) using $m_v$

\[
\text{settlement} = m_v \Delta \sigma \ H
\]

(b) using e-log $\sigma_v'$ plot

\[
\text{settlement} = \frac{\Delta e}{1 + e_o} \ H
\]
Settlement computations

~ computing $\Delta e$ using $e$-log $\sigma_v$’ plot

If the clay is normally consolidated,

the entire loading path is along the VCL.

$$\Delta e = C_c \log \frac{\sigma_v' + \Delta \sigma'}{\sigma_v'}$$

If the clay is overconsolidated, and remains so by the end of consolidation,

$$\Delta e = C_r \log \frac{\sigma_v' + \Delta \sigma'}{\sigma_v'}$$

note the use of $C_r$
Settlement computations

~ computing $\Delta e$ using $e$-$\log \sigma_v'$ plot

If an overconsolidated clay becomes normally consolidated by the end of consolidation,

$$\Delta e = C_r \log \frac{\sigma_p'}{\sigma_{vo}'} + C_c \log \frac{\sigma_{vo}'+\Delta \sigma'}{\sigma_p'}$$

**Consolidation Settlement**

Calculation of Consolidation Settlement under a Foundation

$$\Delta \sigma_{av} = \frac{\Delta \sigma_t + 4\Delta \sigma_m + \Delta \sigma_b}{6}$$

$C_c = 0.009 (LL - 10) = 0.27$

$H = 10 \text{ft} = 120 \text{in}$

$e_o = 1.0$

$\sigma_0' = 10 \times 100 + 10 (120 - 62.9) + \frac{10}{2} (110 - 62.9) = 1814 \text{ psf}$

From Boussinesq solution

$$\Delta \sigma_{av} = 248 \text{ psf}$$

$$S_c = \frac{C_c H}{1 + e_o} \log \frac{\sigma_0' + \Delta \sigma_{av}}{\sigma_0'}$$

$$= 0.9 \text{ in}$$
More to come...

**AM I GETTING THROUGH TO YOU?**

**Consolidation Rate**

**Time Rate of Consolidation**

- Primary consolidation
- Secondary compression

Excess pore water pressure at time $t$:

$H_h$ - height of soil (mm)

$\Delta z$ = change in height (mm)

$\Delta u$ = change in excess pore water pressure (kPa)
**Consolidation Rate**

**Governing Eqn**

\[
\begin{align*}
\frac{\partial \varepsilon}{\partial z} \, dx \, dy \, dz &= \frac{\partial V}{\partial t} \\
\varepsilon_z = k_i &= -k \frac{\partial h}{\partial z} = -k \frac{\partial u}{\partial z} \\
\frac{\partial V}{\partial t} &= \frac{\partial V}{\partial t} = \frac{\partial (V_s + e V_s)}{\partial t} = \frac{2 V_s}{\partial t} + V_s \frac{\partial e}{\partial t} + e \frac{\partial V}{\partial t} \\
\frac{\partial V}{\partial t} &= \frac{dx \, dy \, dz}{1 + e_0} \frac{\partial e}{\partial t} \\
- \frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} &= \frac{1}{1 + e_0} \frac{\partial e}{\partial t} \\
- \frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} &= - \frac{\alpha u}{1 + e_0} \frac{\partial u}{\partial t} \\
\Rightarrow \quad \frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} 
\end{align*}
\]

\[C_v = \frac{k(1 + e_0)}{\gamma_w \cdot \alpha u} = \frac{k}{\gamma_w m_v}\]
Solution of Consolidation Equation

**Assumptions:**

1. Soil is saturated, isotropic, and homogeneous.
2. Darcy’s law is valid.
3. One-dimensional (vertical) compression and flow.
4. Strains are small.
5. Hydraulic conductivity, \( k \) and compressibility, \( m_v \), are constant in the required pressure range.

### Boundary conditions for a uniform distribution of initial pore water pressures in which double drainage occurs:

\[
t = 0: \Delta u = \Delta u_0 = \Delta \sigma_z, \quad t > 0: \Delta u = 0 ; \quad z = 0: \Delta u = 0 ; \quad z = 2H_d : \Delta u = 0
\]

**FOURIER SERIES**

\[
\Delta u(z,t) = \sum_{m=0}^{\infty} \frac{2(\Delta u_0)}{M} \sin \frac{Mz}{H_d} \exp \left(-M^2 T_V \right)
\]

where \( M = (\pi/2) (2m+1) \) and \( m \) is a positive integer with values from 0 to \( \infty \) and

\[
T_V = \frac{c_v t}{H_d^2}
\]

(time factor)

**DEGREE OF CONSOLIDATION**

\[
U_z = \frac{\Delta u_o - \Delta u_z}{\Delta u_o} = 1 - \sum_{m=0}^{\infty} 2 \sin \frac{Mz}{H_d} \exp \left(-M^2 T_V \right)
\]

**AVERAGE DEGREE OF CONSOLIDATION**

\[
U_z = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp \left(-M^2 T_V \right)
\]
Curve 1
For $U = 0 \sim 60\%$, $T_v = \frac{\pi}{4} \left( \frac{U \%}{100} \right)^2$

For $U > 60\%$, $T_v = 1.781 - 0.933 \log (100 - U\%)$

**Determining $C_v$ (logarithmic method)**
Determining $C_v$ (square root method)

$$C_v = \frac{0.197 H^2}{t_{50}} \text{ cm}^2/\text{s}, \text{ m}^3/\text{year}$$

Secondary Consolidation Settlement

Secondary consolidation

$$\delta_s = H \frac{\Delta e_s}{1 + e_0} \quad C_o \log \frac{t_{ult}}{t_{100}}$$