Seismic Hazard Analysis

Seismic hazard analyses involve the quantitative estimation of ground-shaking hazards at a particular site. Seismic hazards may be analyzed deterministically, as when a particular earthquake scenario is assumed, or probabilistically, in which uncertainties in earthquake size, location, and time of occurrence are explicitly considered.

Identification and Evaluation of Earthquake Sources

- Geologic Evidence
  - Fault Activity
  - Magnitude Indicator
- Tectonic Evidence
- Historical Seismicity
- Instrumental Seismicity

Deterministic Seismic Hazard Analysis

A deterministic seismic hazard analysis (DSHA) involves the development of a particular seismic scenario upon which a ground motion hazard evaluation is based. The scenario consists of the postulated occurrence of an earthquake of a specified size occurring at a specified location. A typical DSHA can be described as a four-step process consisting of:

1. Identification and characterization of all earthquake sources capable of producing significant ground motion at the site.
2. Selection of a source to site distance parameter for each source zone.
3. Selection of the controlling earthquake, generally expressed in terms of some ground motion parameters.
4. The hazard at the site is formally defined, usually in terms of the ground motions produced at the site by the controlling earthquake.
When applied to structures for which failure could have catastrophic consequences, such as nuclear power plants and large dams, DSHA provides a straightforward framework for evaluation of worst-case ground motions. However, it provides no information on the likelihood of occurrence of the controlling earthquake, the likelihood of it occurring where it is assumed to occur, the level of shaking that might be expected during a finite period of time (such as the useful lifetime of a particular structure or facility), or the effects of uncertainties in the various steps required to compute the resulting ground motion characteristics.
Probabilistic Seismic Hazard Analysis

In the past 20 to 30 years the use of probabilistic concepts has allowed uncertainties in the size, location, and rate of recurrence of earthquakes and in the variation of ground motion characteristics with earthquake size and location to be explicitly considered in the evaluation of seismic hazards. The probabilistic seismic hazard analysis (PSHA) can also be described as a procedure of four steps, each of which bear some degree of similarity to the steps of the DSHA procedure.

1. Identification and characterization of earthquake sources, is identical to the first step of the DSHA, except that the probability distribution of potential rupture locations within the source must be also be characterized.

2. The seismicity or temporal distribution of earthquake recurrence must be characterized. A recurrence relationship, which specifies the average rate at which an earthquake of some size will be exceeded, is used to characterize the seismicity of each source zone.

3. The ground motion produced at the site by earthquake at the site by earthquakes of any possible size occurring at any possible point in each source zone must be determined with the use of predictive relationships. The uncertainty inherent in the predictive relationship is also considered in a PSHA.

4. Finally, the uncertainties in earthquake location, earthquake size, and ground motion parameter prediction are combined to obtain the probability that ground motion parameter will be exceeded during a particular time period.
Figure 4.6 Four steps of a probabilistic seismic hazard analysis.
Step 1: (Spatial Uncertainty)

Figure 4.7 Examples of different source zone geometries: (a) short fault that can be modeled as a point source; (b) shallow fault that can be modeled as a linear source; (c) three-dimensional source zone.

In most cases, uniform probability distributions are assigned to each source zone, implying that earthquakes are equally likely to occur at any point within the source zone. These distributions are then combined with the source geometry to obtain the corresponding probability distribution of source-to-site distance.

(a) Point source

\[ f_R(r) = 1 \text{ when } r = r_s, \text{ otherwise } f_R(r) = 0 \]

(b) Area source

\[ f_L(l)dl = f_R(r)dr \Rightarrow f_R(r) = f_L(l) \frac{dl}{dr}, \text{ where } f_L(l) = \frac{1}{L_f} \]

Since \( l^2 = r^2 + r_{min}^2 \), \[ f_R = \frac{r}{L_f \sqrt{r^2 - r_{min}^2}} \]

(c) Volumetric source

For source zones with more complex geometries, it is easier to evaluate \( f_R(r) \) by numerical rather than analytical method.
Step 2. (Size Uncertainty and Temporal Uncertainty)

Temporal Uncertainty:

The temporal occurrence of earthquakes is most commonly described by a Poisson model. The Poisson model provides a simple framework for evaluating probabilities of events that follow a Poisson process, one that yields values of a random variable describing the number of occurrences of a particular event during a given time interval. Poisson point processes possess the following properties:

1. The number of point (occurrence) \( N(t_1, t_2) \) in the interval \([t_1, t_2]\) is a Poisson Random Variable with parameter (mean) \( \lambda(t_2 - t_1) \).

\[
P(\{N(t_2 - t_1) = n\}) = e^{-\lambda(t_2 - t_1)} \frac{[\lambda(t_2 - t_1)]^n}{n!} \text{ where } \lambda > 0 \text{ is a constant}
\]

2. If \([t_1, t_2] \cap [t_3, t_4] = \phi\), then \( N(t_1, t_2) \) and \( N(t_3, t_4) \) are statistically independent (for \( t_1 < t_2 < t_3 < t_4 \))

The random process \( N(t) = N(0,t) \) corresponding to a set of Poisson points is called a Poisson (counting) process.

\[
P(\{N = n\}) = e^{-\lambda t} \frac{[\lambda t]^n}{n!}
\]

where \( \lambda \) is the average rate of occurrence of the event and \( t \) is the time period of interest. Note that the probability of occurrence of at least one event in a period time \( t \) is given by

\[
P(N \geq 1) = 1 - P(N = 0) = 1 - e^{-\lambda t}
\]

When the event of interest is the exceedance of a particular earthquake magnitude, the Poisson model can be combined with a suitable recurrence law to predict the probability at least one exceedance in a period of \( t \) years by the expression

\[
P(N \geq 1) = 1 - e^{-\lambda_m t}, \text{ where } \lambda_m \text{ is the mean annual rate of exceedance of an earth quake of magnitude } m.
\]
Size Uncertainty:
Once an earthquake source is identified and its corresponding source zone characterized, the seismic hazard analyst’s attention is turned toward evaluation of the sizes of earthquakes that the source zone can be expected to produce. When the logarithm of the annual rate of exceedance was plotted against earthquake magnitude, a linear relationship was observed. The Gultenberg-Richter law for earthquake recurrence was expressed as

\[ \log \lambda_m = a - bm \quad \text{or} \quad \lambda_m = \exp(\alpha - \beta m) \]

![Diagram](image_url)  
**Figure 4.10** (a) Gutenberg–Richter recurrence law, showing meaning of \( a \) and \( b \) parameters; and (b) application of Gutenberg–Richter law to worldwide seismicity data. (After Esteva, 1970.)
The standard Gutenberg-Richter law covers an infinite range of magnitudes. For engineering purposes, the effects of very small earthquakes are of little interest and it is common to disregard those that are not capable of causing significant damage. If earthquakes smaller than a lower threshold magnitude $m_0$ are eliminated, the mean and annual rate of exceedance can be written as

$$\lambda_m = \lambda_{m_0} \exp[-\beta (m - m_0)], \quad m > m_0$$

The resulting probability distribution of magnitude for the Gutenberg-Richter law with lower bound can be expressed in terms of the CDF:

$$F_M(m) = P[M < m \mid M > m_0] = \frac{\lambda_{m_0} - \lambda_m}{\lambda_{m_0}} = 1 - e^{-\beta (m - m_0)} \quad \text{or the PDF:}$$

$$f_M(m) = \frac{d}{dm} F_M(m) = \beta e^{-\beta (m - m_0)}$$

In general, the source zone will produce earthquakes of different sizes up to some maximum earthquake. If it is known or can be estimated, the mean annual rate of exceedance can be expressed as

$$\lambda_m = \lambda_{m_0} \frac{\exp[-\beta (m - m_0)] - \exp[-\beta (m_{\text{max}} - m_0)]}{1 - \exp[-\beta (m_{\text{max}} - m_0)]}, \quad m_0 \leq m \leq m_{\text{max}}$$

The CDF and PDF for Gutenberg-Richter law with upper and lower bounds can be expressed as

$$F_M(m) = P[M < m \mid m_0 \leq m \leq m_{\text{max}}] = \frac{1 - \exp[-\beta (m - m_0)]}{1 - \exp[-\beta (m_{\text{max}} - m_0)]}$$

$$f_M(m) = \frac{d}{dm} F_M(m) = \frac{\beta \exp[-\beta (m - m_0)]}{1 - \exp[-\beta (m_{\text{max}} - m_0)]}$$
Step 3. (Predictive Relationships)

![Graph showing conditional probability](image)

Figure 4.14  Schematic illustration of conditional probability of exceeding a particular value of a ground motion parameter for a given magnitude and distance.

Step 4. (Combined Uncertainty)

Seismic hazard curves can be obtained for individual source zones and combined to express the aggregate hazard at a particular site. The probability of exceeding a particular value, \( y^* \), of a ground motion parameter, \( Y \), is calculated for one possible earthquake at one possible source location and then multiplied by the probability that that particular magnitude earthquake would occur at that particular location. The process is then repeated for all possible magnitudes and locations with the probabilities of each summed.

For a given \( M > m_0 \) earthquake occurrence, the probability that a ground motion parameter \( Y \) will exceed a particular value \( y^* \) can be computed using the total probability theorem, that is

\[
P[Y > y^*] = \int_{m_0}^{\infty} \int_{-\infty}^{\infty} P[Y > y^* | (M, R) = (m, r)] f_{M,R}(m, r) \, dm \, dr
\]

Assuming that \( M \) and \( R \) are independent, the probability of exceedance can be written as

\[
P[Y > y^*] = \int_{-\infty}^{\infty} \int_{m_0}^{\infty} P[Y > y^* | m, r] f_M(m) f_R(r) \, dm \, dr
\]
If the site of interest is in a region of $N_s$ potential earthquake sources, each of which has an average rate of threshold magnitude exceedance, $\lambda_{m0}$, the total average exceedance rate for the region will be given by

$$\lambda_Y = \sum_{i=1}^{N_s} \lambda_{m0} \left[ \int_{-\infty}^{y^*} \int_{-\infty}^{y^*} P[Y > y^* | m, r] f_M(m) f_R(r) dmdr \right]$$

The seismic hazard curve can easily be combined with the Poisson model to estimate probabilities of exceedance in finite time intervals. The probability of exceedance $y^*$ in a time period $T$ is

$$P(Y_T > y^*) = 1 - e^{-\lambda_Y T}$$