Homework Assignment #4

1. The space \( \mathcal{S} \) consists of all points \( T_i \) in the interval \((0,1)\), and \( P\{0 \leq T_i \leq y\} = y \), for every \( y \leq 1 \). A function \( G(y) \) is increasing from \( G(-\infty) = 0 \) to \( G(\infty) = 1 \); hence it has an inverse \( G^{-1}(y) = H(y) \). A random variable is such that \( Z = H(T_i) \). Show that \( F_Z(x) = G(x) \).

2. Consider the result of the previous problem. Now suppose you have a random number generator that generate random variables with pdf.

\[ f_Z(x) = 1_{(0,1)}(x) \]

and suppose you want to generate a random variable \( Z \) with pdf \( f_Z(z) \). How would you process the output of the uniform random number generator, \( X \), to do this?

3. A fair coin is tossed three times, and the RV \( Z \) equals the total number of heads. Find and sketch \( F_Z(x) \) and \( f_Z(x) \).
4. Show that

\[ F_x(x | A) = \frac{P(A | x \leq x) F_x(x)}{P(A)} \]

5. The probability of heads of a random coin is an RV, \( P \)
   uniform in the interval \((0, 1)\). (a) Find \( P \{0.3 \leq P \leq 0.7\} \)
   (b) The coin is tossed 10 times and heads shows 6 times, find the a posterior probability that \( P \) is between 0.3 and 0.7.

6. Find \( f_Y(y) \) and \( f_Y(Y) \) if \( Y = -4X + 3 \) and \( \sigma_X = 2e^{2x} \), \( \mu_X = 1_{[0, \infty)}(x) \)

7. (a) Show that if \( Y = aX + b \), then \( \sigma_Y = |a| \sigma_X \)
   (b) Find \( \mu_Y \) and \( \sigma_Y \) if \( Y = \frac{X - \mu_x}{\sigma_x} \)

8. Show that if \( A = \{A_1, A_2, \ldots, A_n\} \) is a partition of \( \Omega \), then
   \[ E\{X\} = E\{X | A_1\} P(A_1) + \cdots + E\{X | A_n\} P(A_n) \]