Homework Assignment #2

1. Express each of the following events in terms of the events A, B, and C and the operations of complementation, union, and intersection.

(a) at least one of the events A, B, C occurs
(b) at most one of the events A, B, C occurs
(c) none of the events A, B, C occurs
(d) all three events occur.
(e) exactly one of the events occurs
(f) A and B occur, but not C
(g) A occurs, if not then B does not occur either.

2. Let $\Omega$ be the sample space corresponding to the random experiment of tossing a coin three times, and noting the sequence of heads and tails. Let A be the event that heads occurs exactly twice, let B be the event that at least two heads appear, let C be the event that heads appears when tails has appeared at least once.

(a) Give the elements of A, B, and C.
(b) Describe the events: (i) $\overline{A} \cap B$, (ii) $\overline{A} \cap \overline{B}$, (iii) $A \cap C$
3. If \( S = \{1, 2, 3, 4\} \), find the smallest field contains the sets \( \{1\}, \{2, 3\} \).

4. Show that if \( A \cap B = \emptyset \), then \( P(A) \leq P(B) \).

5. Prove and generalize the following identity

\[
P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)
\]