RELIABILITY-BASED METHOD FOR ASSESSING LIQUEFACTION POTENTIAL OF SOILS

By C. Hsein Juang,1 David V. Rosowsky,2 and Wilson H. Tang3

ABSTRACT: This paper describes a probabilistic method for assessing the liquefaction potential of sandy soils. The proposed probabilistic method is formulated based on the results of reliability analyses of 225 field records, observations of soil performance against liquefaction. The results of the present study show that a meaningful mapping between notional probability and an actual relative frequency measure of the occurrence of liquefaction can be obtained with the proposed method. Twenty case records from the 1989 Loma Prieta earthquake are further analyzed to demonstrate the proposed reliability-based method. The developed method has the potential of becoming a practical tool for engineers involved in the assessment of liquefaction potential.

INTRODUCTION

Because of the difficulty and cost associated with obtaining high quality undisturbed samples, simplified methods based on in-situ tests such as the standard penetration test (SPT), the cone penetration test (CPT), the shear-wave velocity test (V_s), and the Becker penetration test (BPT) are preferred by geotechnical engineers for evaluating the liquefaction potential of soils. In fact, the “simplified procedure” based on the SPT for evaluation of liquefaction potential, developed by Seed and his coworkers (Seed and Idriss 1971, 1982), has become the standard of practice in North America and throughout much of the world. While the SPT-based method is most widely used, the CPT-based methods have become quite popular recently (Olsen and Koester 1995; Olsen 1997; Robertson and Wride 1997; Starks and Olson 1995; Suzuki et al. 1995).

The simplified methods all rely on some “limit states” that separate the liquefaction region from the nonliquefaction region. These limit states are generally established empirically and with a built-in conservatism based on field observations of soil performance against liquefaction in an earthquake at sites where in-situ test data are available. This empirical process in the development of a limit state inevitably induces many uncertainties. To complement the deterministic approaches, statistical and probabilistic methods have been used to deal with these uncertainties. For example, Haldar and Tang (1979) performed second-moment analyses of the SPT-based limit state established by Seed and Idriss (1971). Christian and Swiger (1975) presented statistics of liquefaction and SPT results. Liao et al. (1988) used statistical regression methods to quantify the probability of liquefaction as a function of given earthquake load and SPT resistance. An update on the topic of probabilistic analysis was presented by Yound and Noble (1997).

Most civil engineering systems for which reliability-based design methods have been employed are “one-of-a-kind” (i.e., a dam, building structure, or nuclear power plant, all unique design) and, therefore, it is not possible to build up a sufficient “failure/nonfailure” database. A notional probability of failure can be defined as that value calculated using probabilistic techniques such as second-moment methods (Ditlevsen 1981). As a result of modeling assumptions (both physical and statistical), system idealizations, and incomplete or uncertain information, a notional failure probability will not necessarily be the same as the actual failure probability, i.e., as would be observed (or expected) in the field. This implies notional failure probabilities can be interpreted only in a comparative, rather than a relative frequency, sense. The present inability to relate notional probabilities of failure to those observed or expected in practice has been a major obstacle to the acceptance of reliability methods by practicing engineers. In this study, a new reliability-based method is developed for assessing liquefaction potential. The proposed approach, based on the results of reliability analyses of 225 liquefaction/nonliquefaction case records, enables a more realistic mapping between the notional probability and a true relative frequency measure of the occurrence of liquefaction.

PROPOSED RELIABILITY-BASED METHOD

Reliability Index and Its Distribution

Advanced first-order second-moment (AFOSM) techniques are used to calculate the reliability index in this study. Specifically, the Hasofer-Lind reliability index is computed as follows (Ditlevsen 1981):

$$\beta = \min_{m \in F} \sqrt{(X - m)^{T}C^{-1}(X - m)}$$

(1)

where $X = $ vector of random variables in the limit state function given by $G(X) = 0; m =$ vector of mean values; and $C =$ covariance matrix.

The minimization in (1) is performed over the failure domain $F$ corresponding to the region $G(X) < 0$. A number of numerical techniques have been used to solve this minimization problem. The ellipsoid method (Low 1996; Low and Tang 1997) is used here to perform the minimization and determine the reliability index $\beta$. The advantages of this approach, particularly when implemented in a spreadsheet, are discussed in the literature (Low 1996). Among the practical advantages of this method are: (1) the solution can be obtained by working in original, rather than transformed or reduced random variable space; (2) it is not necessary to provide or calculate partial derivatives of $G(X)$; and (3) correlated and nonnormal variables are handled easily through transformations (Ditlevsen 1981).

For a reliability analysis of the liquefaction potential, the limit state may be written as $G(X) = CRR/CSR - 1 = 0$, where CSR is the cyclic stress ratio that represents loading imposed by an earthquake, and CRR is the cyclic resistance ratio that represents the liquefaction resistance of soil. The term CRR was first proposed by Robertson and endorsed by the National Center for Earthquake Engineering Research (NCEER) Workshop on Evaluation of Liquefaction Resistance of Soils (Yound 1988).
and Idrris 1997). In the present study, CSR is calculated using Seed and Idrris’ (1971, 1982) formulation:

\[
\text{CSR} = 0.65(a_{\text{max}}/g)(\sigma_e/\sigma_v)(r_g/F_{\text{MS}})
\]  

(2)

where \(a_{\text{max}}\) = peak horizontal ground acceleration generated by the earthquake; \(g\) = acceleration of gravity; \(\sigma_v\) = total vertical overburden stress; \(\sigma_e'\) = effective vertical overburden stress; \(r_g\) = stress reduction coefficient; and \(F_{\text{MS}}\) = magnitude scaling factor.

The term \(r_g\) gives an approximate correction for the flexibility of the soil profile. Seed and Idrris (1971) provided a chart showing the mean and the range of \(r_g\) values versus depth. Liao and Whitman (1986) proposed an excellent approximation to the Seed and Idrris mean \(r_g\) values. The latter method, which was adopted during the NCEER workshop (Youd and Idrris 1997), is used in the present study. Note that the certainty with which CSR can be calculated decreases with depth when using mean \(r_g\) values in the calculations (Youd and Idrris 1997). Since almost all field liquefaction data available are at shallow depths where the uncertainty is smaller, \(r_g\) is treated as a nonrandom variable, albeit it is a function of depth.

The term \(F_{\text{MS}}\) is used to correct the calculated CSR for earthquakes with magnitudes smaller or larger than 7.5. This variable is a function of earthquake magnitude \(M\). This term is required, as Seed and Idrris’s simplified method was originally developed for an earthquake magnitude of 7.5. In this study, Idrris’s new formula \(F_{\text{MS}} = 10^{2.55/M^2.56}\) [cited in Youd and Idrris (1997)] is adopted. With this formula, (2) can be rewritten as follows:

\[
\text{CSR} = (0.65/10^{2.55})(a_{\text{max}}/g)(\sigma_e/\sigma_v)(r_g)(M^{2.56})
\]  

(3)

The variables \(a_{\text{max}}, M, \sigma_v\) and \(\sigma_e'\) in (3) are treated as random variables in the present study. These random variables are all assumed to follow normal distribution. In this study, the reported values of these variables are taken as the means. The coefficients of variation (COV) of the variables \(a_{\text{max}}, M, \sigma_v\) and \(\sigma_e'\) are assumed to be 0.15, 0.05, 0.10, and 0.15, respectively. Note that the COV for \(a_{\text{max}}\) is based on the actual measurement in the case records and ranges from 0.06 to 0.29 in the database analyzed. The COV for \(M\) is also based on measurements, although it is based on the reported standard deviation of the measured magnitude in a single event—the 1995 Hyogo-Ken Nanbu Earthquake (Comartin et al. 1995). The probabilistic analyses presented here are concerned only with case records, and thus these COV levels for \(a_{\text{max}}\) and \(M\) are considered appropriate. The assessment of COV levels in \(a_{\text{max}}\) and \(M\) should not be confused with a general liquefaction risk analysis where the seismic loads are also considered as uncertain, in which case the COV of \(a_{\text{max}}\) could easily reach 0.50 or higher (Haldar and Tang 1979) due to uncertain attenuation. The COV for \(\sigma_v\) is estimated based on the fact that the unit weight of soils normally falls in the 15–21 kN/m² range, and thus the COV may be estimated as 0.10, considering the range as the mean plus minus two standard deviations. The COV for \(\sigma_e'\) is considered to be somewhat greater than that for \(\sigma_v\) because of the uncertainty in the ground-water table. A COV of 0.15 for \(\sigma_e'\) is considered appropriate. As with any geotechnical projects, the responsibility of assessing input uncertainties rests with the engineer, and if the perceived uncertainties in terms of COVs differ significantly from the foregoing COV values, some adjustment to the solution obtained in the probabilistic analysis is warranted.

The CRR may be calculated based on SPT-based methods (Seed et al. 1985) or the CPT-based methods. In this study, the CPT-based method proposed by Robertson and Wride (1997) is used as an example to illustrate the proposed methodology. The original Robertson and Wride method involves several empirical correction steps and is quite complicated. Here, an approximation by means of an artificial neural network (ANN) model is used. This ANN model (Juang et al. 1999), a three-layer neural network, yields practically the same results as those by using the Robertson and Wride (1997) method. The model yields a CRR value for a given set of input variables—\(q_e, f_s\), \(r_p\) and \(\rho_c\). The first two variables, cone tip resistance \(q_e\) and sleeve friction \(f_s\) are CPT measurements. In this study, the COV’s for input variables \(q_e\) and \(f_s\) are assumed to be 0.15 and 0.20, respectively. These values are based on statistical analysis of CPT data by (Kulhawy and Trautmann (1996)) with minor adjustments based on Juang and Tso’s sensitivity study (1998). The COV for CRR model uncertainty is assumed to be 0.10, which is estimated based on the standard error of the CRR prediction as deviated from those obtained by the Robertson and Wride method (1997).

The correlation among the input variables also needs to be considered in the reliability analysis. There is strong correlation between \(\sigma_v\) and \(\sigma_e'\) and between \(a_{\text{max}}\) and \(M\), while the correlations for all other pairs of variables are weak. The correlation coefficient is obtained by using standard statistical methods (Mendenhall and Sinich 1995). Based on available data, the correlation coefficient between \(\sigma_v\) and \(\sigma_e'\) is determined to be about 0.95, and a correlation coefficient between \(a_{\text{max}}\) and \(M\) is determined to be about 0.90. These values are used in this paper, while all other variables are assumed to be independent.

The present reliability analysis is carried out using (1). The database, consisting of 131 liquefied cases and 94 non liquefied cases, is taken from the literature (Arulanandan et al. 1986; Bennett et al. 1981; Bennett et al. 1984; Bennett 1989; Bennett and Tinsley 1995; Bierschweiler and Stokoe 1984; Boulanger et al. 1997; Kayen et al. 1992; Kayen et al. 1998; Stark and Olson 1995; Tinsley et al. 1998; USGS 1990; Wayne et al. 1998). For each case, the tabulated data include depth of the water table, depth of the liquefaction observation, and the six input variables—\(a_{\text{max}}, M, q_e, f_s\) and \(\rho_c\). Here, the reported values of the six input variables are taken as the mean values, the aforementioned COV values are used, and all random variables are assumed to be normally distributed. The reliability index \(\beta\) is calculated with the following equation, derived from (1) and based on the foregoing correlation assumption:

\[
\beta^2 = \min_{\text{COVs}=0} \left\{ \frac{(x_i - m_i)^2}{\sigma_i^2} + \frac{(x_j - m_j)^2}{\sigma_j^2} \right\}
\]

where \(x_i = \text{random variable } q_e; x_j = \text{random variable } f_s; x_3 = \text{random variable } \sigma_v; x_4 = \text{random variable } \sigma_e'; x_5 = \text{random variable } a_{\text{max}}; x_6 = \text{random variable } M; m_i = \text{mean value of random variable } x_i (i = 1, 7); \sigma_i = \text{standard deviation of random variable } x_i; \rho_{ij} = \text{correlation coefficients between } x_i \text{ and } x_j \text{ and } x_6.\) The reliability index \(\beta\) is determined from (4) subject to the constraint of \(G(X) = 0\). The calculated \(\beta\) values are grouped according to whether or not liquefaction actually occurred at the site. Fig. 1 shows the results of these calculations for all cases. There is a considerable overlap between the liquefied cases (denoted as Group L), and the non-
liquefied cases (denoted as Group NL). An evaluation of the distribution of β values reveals that both Group L and Group NL could be well fit using the Rayleigh distribution, defined as

\[ f(\beta) = \frac{1}{\beta} \cdot \beta e^{-\frac{\beta^2}{2\beta^2}} \]

where \( b \) and \( t \) = parameters that define the Rayleigh distribution. For liquefied cases (Group L data), curve-fitting yields: \( b = 3.44 \) and \( t = 3.14 \) (Fig. 1). For nonliquefied cases (Group NL data), curve-fitting yields: \( b = 1.75 \) and \( t = 4.93 \). These distributions are adopted in the present study. As more data become available, the choice of distribution functions can be further investigated.

**Probability of Liquefaction**

For a case in which a reliability index \( \beta \) has been calculated, the probability that liquefaction will occur would be:

\[ P(L|\beta) = \frac{P(\beta)L|P(L)P(\beta|L) + P(\beta|NL)P(\beta|NL)}{P(L)P(\beta|L) + P(\beta|NL)P(\beta|NL)} \]

where \( P(L|\beta) \) = probability of liquefaction for a given \( \beta \); \( P(\beta|L) \) = distribution function of \( \beta \), given that liquefaction did occur; \( P(\beta|NL) \) = distribution function of \( \beta \), given that liquefaction did not occur; \( P(L) \) = prior probability of liquefaction; and \( P(\beta) \) = prior probability of no liquefaction. Note that \( P(\beta|L) \) and \( P(\beta|NL) \) may be determined empirically based on sufficient field observations, as those distributions of \( \beta \) shown in Fig. 1. Thus

\[ P(\beta|L) = \int_{\beta}^{\beta+\Delta\beta} f_L(x) \, dx \] \hspace{1cm} (7a)

and

\[ P(\beta|NL) = \int_{\beta}^{\beta+\Delta\beta} f_{NL}(x) \, dx \] \hspace{1cm} (7b)

where \( f_L(x) \) and \( f_{NL}(x) \) = probability density functions of \( \beta \) for Groups L and NL, respectively [Fig. 1 and (5)]. As \( \Delta \beta \to 0 \), (6) can be simply expressed as follows:

\[ P(L|\beta) = \frac{f_L(\beta)P(L)}{f_L(\beta)P(L) + f_{NL}(\beta)P(NL)} \]

(8)

If knowledge of prior probabilities \( P(L) \) and \( P(NL) \) is available, (8) can be used to determine the probability of liquefaction for a given \( \beta \). In the absence of such knowledge, it may be assumed, based on the principle of maximum entropy (Jaynes 1957, 1979), that \( P(L) = P(NL) \). Under the assumption that \( P(L) = P(NL) \), (8) can be expressed as follows:

\[ P(L|\beta) = \frac{f_L(\beta)}{f_L(\beta) + f_{NL}(\beta)} \]

(9)

In the analysis presented here, (9) is used to determine the probability of liquefaction for all field cases with calculated \( \beta \) values. The results are shown in Fig. 2 along with the notional probability calculated using second-moment methods, \( P_f = \Phi(\beta) \). The function \( P(L|\beta) \) relates the reliability index \( \beta \) to the actual probability of liquefaction based on the field observations. The actual probability of liquefaction is seen to be lower than the notional probability at a given \( \beta \). For example, at \( \beta = 0 \), the notional probability \( P_f = \Phi(\beta) = 0.5 \), while the actual probability \( P(L|\beta) = 0.16 \). This is certainly reasonable as it is always inferred that notional probabilities are higher than actual values owing to redundancies, system interaction, and other sources of conservatism not explicitly accounted for in the reliability analysis. In the writers' opinion the main reason for this discrepancy is that the adopted limit state, in terms of the CSR [Eq. (3)] and CRR models [Robertson and Wride method (1997)], has a considerable degree of built-in conservatism. Thus, if the foregoing conservative models are used in the calculations for CSR and CRR, the probability of liquefaction should be taken as \( P(L|\beta) \), rather than the notional probability, \( P_f = \Phi(\beta) \).

**CASE STUDY**

The 1989 Loma Prieta earthquake caused extensive liquefaction at several locations within the Moss Landing area, located on Monterey Bay in California. Details on the locations and extent of liquefaction in this event at Moss Landing have been documented elsewhere [for example, Boulanger et al. (1997)]. In the present study, 20 case records from Moss Landing (data were obtained from published literature as well as from Dr. Leslie Youd, of Brigham Young University, through private communication, 1998) are analyzed. For each case, the CRR and CSR are calculated, along with the reliability index and probability of liquefaction. Table 1 shows a summary of this reliability analysis for all cases at the depths where soil performance against liquefaction was reported. For each of these cases, the CSR, CRR, and the probability of liquefaction (\( P_L \)) are calculated continuously at all depths so that a profile of \( P_L \) can be plotted. Fig. 3 shows a sample output of the \( P_L \) profile, along with the CSR and CRR profiles and the input CPT profiles.

Plots of the CSR and CRR profiles, such as those shown in Fig. 3(c), are quite useful, as they show which layers are likely to liquefy. However, this assessment of the liquefaction potential is essentially deterministic. Because of the uncertainties involved in the calculation of CSR and CRR, such a deter-
TABLE 1. Summary of Reliability Analyses of Moss Landing Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Depth (m)</th>
<th>$q_o$ (MPa)</th>
<th>$f_s$ (kPa)</th>
<th>$\sigma_v$ (kPa)</th>
<th>$\sigma_v'$ (kPa)</th>
<th>$\alpha_{max}$ (g)</th>
<th>$M$ (8)</th>
<th>CRR (9)</th>
<th>CSR (10)</th>
<th>FS</th>
<th>$\beta$ (12)</th>
<th>$P_c$ (13)</th>
<th>Liquefaction observation (14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC-1</td>
<td>11</td>
<td>4.60</td>
<td>46</td>
<td>207.7</td>
<td>129.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.11</td>
<td>0.20</td>
<td>0.6</td>
<td>2.23</td>
<td>0.783</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-2</td>
<td>1.9</td>
<td>10.40</td>
<td>33.28</td>
<td>41.4</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.41</td>
<td>0.15</td>
<td>2.8</td>
<td>3.84</td>
<td>0.000</td>
<td>No</td>
</tr>
<tr>
<td>UC-3</td>
<td>2.5</td>
<td>8.70</td>
<td>30.45</td>
<td>45.1</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.31</td>
<td>0.16</td>
<td>2.0</td>
<td>2.46</td>
<td>0.003</td>
<td>No</td>
</tr>
<tr>
<td>UC-4</td>
<td>2.5</td>
<td>7.70</td>
<td>20.79</td>
<td>45.1</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.25</td>
<td>0.16</td>
<td>1.6</td>
<td>1.61</td>
<td>0.013</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-5</td>
<td>2.5</td>
<td>7.70</td>
<td>20.79</td>
<td>45.1</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.56</td>
<td>0.21</td>
<td>2.7</td>
<td>4.05</td>
<td>0.000</td>
<td>No</td>
</tr>
<tr>
<td>UC-6</td>
<td>6.5</td>
<td>18.20</td>
<td>30.94</td>
<td>120.4</td>
<td>76.9</td>
<td>0.25</td>
<td>7.1</td>
<td>0.54</td>
<td>0.21</td>
<td>2.6</td>
<td>3.95</td>
<td>0.000</td>
<td>No</td>
</tr>
<tr>
<td>UC-7</td>
<td>8.3</td>
<td>4.30</td>
<td>77.4</td>
<td>156.7</td>
<td>94.3</td>
<td>0.25</td>
<td>7.1</td>
<td>0.15</td>
<td>0.22</td>
<td>0.7</td>
<td>1.42</td>
<td>0.545</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-8</td>
<td>8.6</td>
<td>4.30</td>
<td>55.9</td>
<td>160.2</td>
<td>96.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.12</td>
<td>0.22</td>
<td>0.6</td>
<td>2.15</td>
<td>0.763</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-9</td>
<td>2.9</td>
<td>6.60</td>
<td>19.8</td>
<td>53.7</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.20</td>
<td>0.19</td>
<td>1.0</td>
<td>0.11</td>
<td>0.130</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-10</td>
<td>2.2</td>
<td>3.10</td>
<td>12.4</td>
<td>50.5</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.10</td>
<td>0.18</td>
<td>0.6</td>
<td>2.11</td>
<td>0.752</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-11</td>
<td>2.2</td>
<td>3.10</td>
<td>11.16</td>
<td>53.2</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.10</td>
<td>0.19</td>
<td>0.5</td>
<td>2.34</td>
<td>0.811</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-12</td>
<td>4.1</td>
<td>6.20</td>
<td>55.8</td>
<td>77.1</td>
<td>54.9</td>
<td>0.25</td>
<td>7.1</td>
<td>0.19</td>
<td>0.19</td>
<td>1.0</td>
<td>0.07</td>
<td>0.162</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-13</td>
<td>4.1</td>
<td>4.30</td>
<td>43</td>
<td>77.1</td>
<td>54.9</td>
<td>0.25</td>
<td>7.1</td>
<td>0.14</td>
<td>0.19</td>
<td>0.7</td>
<td>1.27</td>
<td>0.495</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-14</td>
<td>3</td>
<td>3.80</td>
<td>19</td>
<td>52.6</td>
<td>42.7</td>
<td>0.25</td>
<td>7.1</td>
<td>0.12</td>
<td>0.17</td>
<td>0.7</td>
<td>1.47</td>
<td>0.562</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-15</td>
<td>3</td>
<td>3.00</td>
<td>12</td>
<td>52.6</td>
<td>42.7</td>
<td>0.25</td>
<td>7.1</td>
<td>0.10</td>
<td>0.17</td>
<td>0.6</td>
<td>2.11</td>
<td>0.750</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-16</td>
<td>2.3</td>
<td>6.60</td>
<td>29.04</td>
<td>39.4</td>
<td>39.1</td>
<td>0.25</td>
<td>7.1</td>
<td>0.21</td>
<td>0.14</td>
<td>1.5</td>
<td>1.85</td>
<td>0.008</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-17</td>
<td>4.4</td>
<td>5.40</td>
<td>21.6</td>
<td>77.2</td>
<td>62.0</td>
<td>0.25</td>
<td>7.1</td>
<td>0.12</td>
<td>0.17</td>
<td>0.7</td>
<td>1.33</td>
<td>0.517</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-18</td>
<td>4</td>
<td>16.40</td>
<td>44.28</td>
<td>69.9</td>
<td>64.0</td>
<td>0.25</td>
<td>7.1</td>
<td>0.53</td>
<td>0.15</td>
<td>3.6</td>
<td>5.31</td>
<td>0.000</td>
<td>No</td>
</tr>
<tr>
<td>UC-20</td>
<td>4.7</td>
<td>4.10</td>
<td>23.78</td>
<td>84.9</td>
<td>68.3</td>
<td>0.25</td>
<td>7.1</td>
<td>0.10</td>
<td>0.17</td>
<td>0.6</td>
<td>1.99</td>
<td>0.717</td>
<td>Yes</td>
</tr>
<tr>
<td>UC-21</td>
<td>4.2</td>
<td>4.90</td>
<td>23.52</td>
<td>75.8</td>
<td>61.0</td>
<td>0.25</td>
<td>7.1</td>
<td>0.11</td>
<td>0.17</td>
<td>0.7</td>
<td>1.49</td>
<td>0.566</td>
<td>Yes</td>
</tr>
</tbody>
</table>

FIG. 3. Sample Output of Cyclic Liquefaction Resistance and Probability of Liquefaction

The deterministic approach is not always appropriate. The plot of the $P_c$ profile, as shown in Fig. 3(d), offers an alternative on which engineering decisions may be based. With this profile, the engineer can determine which layers are susceptible to liquefaction from the viewpoint of an acceptable risk level. This advantage is also observed in Table 1. For example, in the case of UC-9 at the depth of 2.9 m, the comparison of calculated CSR and CRR suggests that there would be no liquefaction since CRR > CSR (albeit slightly). However, the field observation indicates the occurrence of liquefaction. The probability of liquefaction for this case is 0.13, which suggests that liquefaction may be possible. Similar observation is found in the case of UC-12. In the case of UC-16, the deterministic method yields a CRR of 0.21 and a CSR of 0.14, which suggests that liquefaction will not occur. However, the field observation indicates the occurrence of liquefaction. For this case, the result of the probability analysis ($P_c = 8 \times 10^{-7}$) does not yield a credible support of the occurrence of liquefaction; thus, the probabilistic analysis does not yield an advantage over the deterministic method.

Fig. 4 shows a plot of the factor of safety (FS)—defined as the simple ratio of CRR over CSR—versus the probability of liquefaction for the 20 Moss Landing cases analyzed. It is observed that the $P_c$ value is very close to 0 if FS is greater than 1.
than 2.0. Thus, a conservative and expensive design with a FS of 2 would almost eliminate the chance of liquefaction, even though it is recognized that the FS approach does not account for parameter and model uncertainties. However, if the FS is between 1.0 and 1.5, the effect of uncertainty on the computed FS becomes more significant—the probability of liquefaction can change drastically for a small change in the FS (Fig. 4).

Another important observation about Fig. 4 is that: Based on statistical analyses by Liao et al. (1988), the Seed and Idriss (1971) SPT-based limit state curve indicates a probability of liquefaction for clean sands of about 20% at FS = 1. Here, Fig. 4 suggests the probability of liquefaction is about 16% at FS = 1. This result is remarkably consistent with that obtained by Liao et al. (1988), although a different type of in-situ test (CPT as opposed to SPT) was used in this study.

If the limit state is accurate and without any built-in conservatism, one would expect the probability of liquefaction to be 0.5 at FS = 1. The limit state adopted here, in terms of the CSR and CRR models, has a considerable degree of conservatism built into it, thus the lower probability of liquefaction at FS = 1 is expected. Since the limit state based on CPT [Robertson and Wride method (1997)] and that based on SPT (Seed and Idriss method) yielded a comparable probability at FS = 1, it might be inferred that the degree of conservatism in the Robertson and Wride method is comparable to that in the Seed and Idriss method. This theory is being investigated by the writers, and the results are expected to be published in the near future.

CONCLUSION

A new framework for the reliability analysis of liquefaction potential has been presented in this paper. Excellent results have been obtained in terms of being able to assess the liquefaction potential in a more rational way. The method has been implemented in a spreadsheet and, given the CPT profiles, the profile of the probability of liquefaction can be easily obtained. This method has the potential of becoming a practical tool for the engineer involved in the assessment of liquefaction potential. The developed spreadsheet modules are available from the writers.

Further work to refine the proposed method is needed, including: (1) better definition of the distributions of \( \beta \) values in both the liquefaction and nonliquefaction groups; (2) investigation of the effect of possible variation in COVs of the input parameters in the probabilistic model; (3) development of guidelines for the selection of FS for design based on an acceptable level of risk (or probability of liquefaction); and (4) combination of estimates of the probability of liquefaction using more than one in-situ test. These issues are being investigated in a research project at Clemson University and observations are expected to be reported in the near future.

ACKNOWLEDGMENTS

The study on which this paper is based is supported by the National Science Foundation (NSF) through Grant No. CMS-9612116. The cognizant NSF program official for this grant is Dr. Clifford Astill. This financial support is greatly appreciated. The writers are in debt to Dr. Leslie Youd of Brigham Young University and Michael Bennett of the U.S. Geological Survey for their kindness in providing CPT data at the historical sites where liquefaction performance was observed. Caroline J. Chen of Clemson University assisted in the reliability analysis and her assistance is appreciated. The journal reviewers are thanked for their constructive comments that help sharpen this paper.

APPENDIX. REFERENCES


