I. Mathematical Fundamentals of Probability Theory

Set Theory
- Basic Definitions
- Set Operations
- Algebra of Sets

Probability Space
A random experiment is characterized by probability space (S, F, P)
- Sample Space
- Event Space
- Probability Measures (Sum Rule)
- Probability Calculus
- Counting
- Equal Likely Outcomes

Conditional Property and Independence
- Conditional Property (Product Rule)
- Independence
- Combined Experement
- Independent Experiments

II. Random Variable & Stochastic Process

Single Random Variable
- Definition
- Cumulative Distribution Function (CDF)
- Probability Density Function (PDF)
- Conditional Distribution
- Function of Random Variable
- Moments of Random Variable

Two Random Variables
- Definition
- Joint CDF
• Joint PDF
• Marginal Distribution
• Conditional Distribution and Statistically Independence
• One Function of Two Random Variables
• Two Functions of Two Random Variables
• Joint Moments

Random Vector
• Definition
• Joint CDF
• Joint PDF
• Marginal Distribution
• Conditional Distribution and Statistically Independence
• Transformation of Random Vectors
• Joint Moments

Probability Distribution and Inequality
• Probability Distributions (Discrete and Continuous)
• Principle of Maximum Entropy
• Statistical/Probabilistic Model
• Random Number Generation
• Descriptive Statistics and Sample Statistics
• Inequality

Simulation of System of Random Variables
• Computation of CDF for Nonlinear Function of Random Variables (Monte Carlo Simulation)
• Nonlinear Computation of Moments of \( Y = g(X) \)
  • Sample Statistics from Monte Carlo Simulation
  • Direct Integration
  • Taylor Series (First Order Second Moment)
  • Point Estimate Method

Stochastic Convergence
• Order Statistics and Stochastic Convergence
• Definitions
• Weak Law of Large Number
• Central Limit Theorem
Stochastic Process
- Definition
- Probabilistic Description of Random Process
- System of Stochastic Process

III. Reliability Analysis
Reliability analysis is basically a problem of system of random variables

IV. Statistical Inference (Bayesian Approach)

Parameter Estimation

Model Selection (Hypothesis)

Assigning Probabilities

Non-parametric Estimation

Experimental Design

V. Statistical Mechanics

Random Walk and Diffusion Process

Ensemble and Ergodic Hypothesis

Analysis of Flow System

Ground Stress Distribution

Ground Settlement