Lateral Earth Pressure Coefficient

- $K = \frac{\sigma_x'}{\sigma_z'}$ = lateral earth pressure coefficient
- $\sigma_x' = \text{horizontal effective stress}$
- $\sigma_z' = \text{vertical effective stress}$
- Ratio of resultant horizontal stress to applied vertical stress
- Similar to Poisson's Ratio for elastic materials

Mohr's Circle and Lateral Earth Pressures

- $\alpha = \frac{\theta}{2}$
- $\tau' = \sigma_1' + \sigma_3' \tan \phi' + c'$
- $\sigma_1' = \sigma_z' - 2\tau' \tan \phi' + c'$
- $\sigma_3' = \sigma_z' + 2\tau' \tan \phi' + c'$

Retaining Walls

- Necessary in situations where gradual transitions either take up too much space or are impractical for other reasons
- Retaining walls are analysed for both resistance to overturning and structural integrity
- Two categories of retaining walls
  - Gravity Walls (Masonry, Stone, Gabion, etc.)
  - In-Situ Walls (Sheet Piling, cast in-situ, etc.)
Groundwater Effects

Conditions of Lateral Earth Pressure Coefficient

- At-Rest Condition
  - Condition where wall movement is zero or “minimal”
  - Ideal condition of wall, but seldom achieved in reality

- Active Condition
  - Condition where wall moves away from the backfill
  - The lower state of lateral earth pressure

- Passive Condition
  - Condition where wall moves toward the backfill
  - The higher state of lateral earth pressure

Groundwater Effects

- Steps to properly compute horizontal stresses including groundwater effects:
  - Compute total vertical stress
  - Compute effective vertical stress by removing groundwater effect through submerged unit weight; plot on $P_o$ diagram
  - Compute effective horizontal stress by multiplying effective vertical stress by $K$
  - Compute total horizontal stress by directly adding effect of groundwater unit weight to effective horizontal stress
Estimates of At Rest Lateral Earth Pressure Coefficient

- Jaky’s Equation
  \[ K_o = 1 - \sin \phi' \]
- Modified for Overconsolidated Soils
  \[ K_o = (1 - \sin \phi') \cdot OCR \cdot \sin \phi' \]
- Applicable only when ground surface is level
- In spite of theoretical weaknesses, Jaky’s equation is as good an estimate of the coefficient of lateral earth pressure as we have

Example of At Rest Wall Pressure

- Given
  - Retaining Wall as Shown
- Find
  - \( P_A \) from At Rest Conditions

Wall Movements Necessary to Achieve Active or Passive States

<table>
<thead>
<tr>
<th>Type of Backfill</th>
<th>Active</th>
<th>Passive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense sand</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>Medium-dense sand</td>
<td>0.002</td>
<td>0.02</td>
</tr>
<tr>
<td>Loose sand</td>
<td>0.004</td>
<td>0.04</td>
</tr>
</tbody>
</table>

\( Y \) = movement of top of wall required to reach minimum active or maximum passive pressure, by tilting or lateral translation.

\( H \) = height of wall.
**Development of Passive Earth Pressure**

\[ K_o = 1 - \sin \phi' \]

\[ K_o = 1 - \sin 30^\circ = 0.5 \]

**At Rest Pressure Example**

- Compute at rest earth pressure coefficient
- Compute Effective Wall Force

\[
\frac{P_o}{b} = \frac{y_1 z_1^2 K_o}{2} \]

\[
\frac{P_o}{b} = \frac{120 \times 20^2 \times 0.5}{2} \]

\[
P_o = \frac{12000 \text{ lbs}}{12 \text{ kips/ft}} = 6.67 \text{ ft}. \]

(valid for all theories)

---

**Earth Pressure Theories**

**Development of Active Earth Pressure**

\( \sigma_o = \text{at-rest pressure} \)
\( \sigma_p = \text{active pressure} \)
Rankine Coefficients with Inclined Backfills

\[
K_A = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}
\]

\[
K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}
\]

Inclined and level backfill equations are identical when \(\beta = 0\)

Example of Rankine Active Wall Pressure

- Given
  - Retaining Wall as Shown
- Find
  - \(P_A\) from At Rest Conditions

Rankine Theory with Inclined Backfills

\[
H = \text{Height of Wall}
\]

\[
\beta = \text{Slope Angle}
\]

For Granular Backfill \(\phi > 0, c = 0\)
**Rankine Active Pressure Example**

- Compute at rest earth pressure coefficient
  \[ K_a = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) \]
  \[ K_a = \tan^2 \left( 45 - 15 \right) = \frac{1}{3} \]
- Compute Effective Wall Force
  \[ \frac{P_o}{b} = \frac{y_1 z_1^2 K_a}{2} \]
  \[ \frac{P_o}{b} = \frac{120 \times 20^2 \times 0.333}{2} \]
  \[ \frac{P_o}{b} = 8000 \text{ lbs/ft} = 8 \text{kips/ft} \]

---

**Rankine Passive Pressure Example**

- Compute at rest earth pressure coefficient
  \[ K_p = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \]
  \[ K_p = \tan^2 \left( 45 + 15 \right) = 3 \]
- Compute Effective Wall Force
  \[ \frac{P_o}{b} = \frac{y_1 z_1^2 K_p}{2} \]
  \[ \frac{P_o}{b} = \frac{120 \times 20^2 \times 3}{2} \]
  \[ \frac{P_o}{b} = 72000 \text{ lbs/ft} = 72 \text{kips/ft} \]

---

**Summary of Rankine and At Rest Wall Pressures**

- Classical earth pressure maximum value for \( \delta = 0 \)
  \[ 72,000 \text{ lbs.} \]
- Classical earth pressure minimum value for \( \delta = \theta \)
  \[ 12,000 \text{ lbs.} \]
  \[ 8000 \text{ lbs.} \]

---

**Rankine Passive Pressure Example**

- Classical earth pressure maximum value for \( \delta = 0 \)
  \[ 120 \text{ pcf} \]
  \[ \phi' = 30^\circ \]
Example of Coulomb Theory

- **Given**
  - Wall as shown above

- **Find**
  - $K_A$, $K_P$, $P_A$

Solution for Coulomb Active Pressures

- **Compute Coulomb Active Pressure**
  
  $K_A = \cos^2(30 - \theta) \cos^2(0 + \phi) \left[ 1 + \sqrt{\frac{\sin(30 + \phi) \sin(30 - \theta)}{\cos(30) \cos(60)}} \right]^2$

  - $K_A = 0.3465$

- **Compute Total Wall Force**
  
  $P_A = 0.3465 \cdot \frac{1}{2} (120 \text{ pcf})(20')^2$

  - $P_A = 8316 \text{ lb/ft of wall}$

Typical Values of Wall Friction

<table>
<thead>
<tr>
<th>Interface Materials</th>
<th>Friction Factor, $\mu$</th>
<th>Friction Angle, $\delta$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass concrete on the following foundation materials:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean sand rock</td>
<td>0.70</td>
<td>15</td>
</tr>
<tr>
<td>Clean gravel, gravel-sand mixtures, coarse sand...</td>
<td>0.55 to 0.60</td>
<td>20 to 31</td>
</tr>
<tr>
<td>Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel...</td>
<td>0.45 to 0.53</td>
<td>24 to 29</td>
</tr>
<tr>
<td>Clean fine sand, silty or clayey fine to medium sand...</td>
<td>0.35 to 0.45</td>
<td>19 to 24</td>
</tr>
<tr>
<td>Very stiff and stiff clay and silty clay...</td>
<td>0.30 to 0.35</td>
<td>17 to 19</td>
</tr>
<tr>
<td>Medium stiff and stiff clay and silty clay...</td>
<td>0.20 to 0.35</td>
<td>17 to 19</td>
</tr>
</tbody>
</table>

(Masonry on foundation materials has same friction factors.)

Steel sheet piling against the following soils: | | |
| Clean gravel, gravel-sand mixtures, well-graded rock fill with spalls... | 0.40 | 22 |
| Clean sand, silty sand-gravel mixture, single size hard rock fill... | 0.30 | 17 |
| Silty sand, gravel or sand mixed with silt or clay | 0.25 | 14 |
| Fine sandy silt, nonplastic silt... | 0.20 | 11 |

Finned concrete or concrete sheet piling against the following soils:

- Clean gravel, gravel-sand mixture, well-graded rock fill with spalls...
- Clean sand, silty sand-gravel mixture, single size hard rock fill...
- Silty sand, gravel or sand mixed with silt or clay
- Fine sandy silt, nonplastic silt...

Various structural materials:

- Masonry on masonry, igneous and metamorphic rocks:
- 0.70 | 35 |
- 0.65 | 33 |
- 0.55 | 29 |
- 0.50 | 26 |
- Masonry on wood (cross grain):
- 0.30 | 17 |
- Steel on steel at sheet pile interlockings...
Solution for Coulomb Passive Pressures

- Compute Coulomb Passive Pressure
  \[ K_p = \cos^2 (30+0) \]
- Compute Total Wall Force
  \[ P_p = 4.0196 \]
- Compute Total Wall Force
  \[ P_p = 96,470 \text{ lb/ft of wall} \]

Walls with Cohesive Backfill

- Retaining walls should generally have cohesionless backfill, but in some cases cohesive backfill is unavoidable.
- Cohesive soils present the following weaknesses as backfill:
  - Poor drainage
  - Creep
  - Expansiveness

Most lateral earth pressure theory was first developed for purely cohesionless soils (c = 0) and has been extended to cohesive soils afterward.

Theory of Cohesive Soils

Rankine Pressures with Cohesion (Level Backfill)

- Overburden Driving
  - Active
    \[ \sigma_A = \sigma_1 \tan^2 (4\frac{\pi}{4} - \frac{\phi}{2}) - 2c \tan (4\frac{\pi}{4} - \frac{\phi}{2}) - \frac{\pi}{4} \]
  - Passive
    \[ \sigma_P = \sigma_3 \tan^2 (4\frac{\pi}{4} + \frac{\phi}{2}) + 2c \tan (4\frac{\pi}{4} + \frac{\phi}{2}) + \frac{\pi}{4} \]

- Wall Driving
  - Active
    \[ \sigma_A = \sigma_1 \tan^2 (4\frac{\pi}{4} - \frac{\phi}{2}) - 2c \tan (4\frac{\pi}{4} - \frac{\phi}{2}) - \frac{\pi}{4} \]
  - Passive
    \[ \sigma_P = \sigma_3 \tan^2 (4\frac{\pi}{4} + \frac{\phi}{2}) + 2c \tan (4\frac{\pi}{4} + \frac{\phi}{2}) + \frac{\pi}{4} \]
Example of Equivalent Fluid Method

- Given
  - Wall as shown above
  - $K_A = 0.3465$
  - $K_p = 4.0196$
  - $\phi_w = 3$ degrees

- Find
  - Forces acting on the wall (both horizontal and vertical)

Comments on Rankine Equations

- Valid if wall-soil friction is not taken into account
- Do not take into consideration soil above critical height
- Do not take into consideration sloping walls
- For practical problems, should use equations as they appear in the book

$H_c = \frac{2c}{\gamma \sqrt{K_a}}$

Example of Equivalent Fluid

- Compute Equivalent Fluid Unit Weights (Active Case)

  \[
  G_h = \gamma K_a \cos \phi_w \\
  G_h = 120 \times 0.3465 \times \cos 3^\circ \\
  G_h = 41.52 \text{ pcf} \\
  G_v = \gamma K_a \sin \phi_w \\
  G_v = 120 \times 0.3465 \times \sin 3^\circ \\
  G_v = 2.18 \text{ pcf}
  \]

Equivalent Fluid Method

- Simplification used to guide the calculations of lateral earth pressures on retaining walls
- Can be used for Rankine and Coulomb lateral earth pressures
- Can be used for at rest, active and passive earth pressures
- Transforms the soil acting on the retaining wall into an equivalent fluid
Example of Equivalent Fluid

- Compute Wall Load (Passive Case)

\[ P_p = \frac{G_h H^2}{2} \]
\[ \frac{P_p}{b} = \frac{481.69 \times 20^2}{2} = 96338 \text{ lb/ft} \]

\[ V_p = \frac{G_v H^2}{2} \]
\[ \frac{V_p}{b} = \frac{25.24 \times 20^2}{2} = 5048 \text{ lb/ft} \]

- Compute Wall Load (Active Case)

\[ P_a = \frac{G_h H^2}{2} \]
\[ \frac{P_a}{b} = \frac{41.52 \times 20^2}{2} = 8304 \text{ lb/ft} \]

\[ V_a = \frac{G_v H^2}{2} \]
\[ \frac{V_a}{b} = \frac{2.18 \times 20^2}{2} = 436 \text{ lb/ft} \]

## Terzaghi Model

- Assumes log spiral failure surface behind wall
- Requires use of suitable chart for K_A and K_p
- Not directly used in this course, but option in SPW 911

Example of Equivalent Fluid

- Compute Equivalent Fluid Unit Weights (Passive Case)

\[ G_h = \gamma K_p \cos \phi_w \]
\[ G_h = 120 \times 4.0196 \times \cos 3^\circ \]
\[ G_h = 481.69 \text{ pcfs} \]

\[ G_v = \gamma K_p \sin \phi_w \]
\[ G_v = 120 \times 4.0196 \times \sin 3^\circ \]
\[ G_v = 25.24 \text{ pcfs} \]
Homework Set 5

- Reading
  - McCarthy: Chapter 16
  - Coduto: Chapters 22, 23, 24 & 25

- Homework Problems
  - McCarthy: 16-1, 16-8, 16-12a, 16-17
  - Coduto: 25.3 (Hand and Chart Solutions); 25.5 (SPW 911)

- Due Date: 17 April 2002

Questions

Surcharge and Groundwater Loads

Figure 3-20  Lateral pressures, one soil, water, finite surcharge