Theoretical model of a multisection time domain reflectometry measurement system

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Abstract. A multisection model is presented to simulate electromagnetic wave propagation in an unmatched time domain reflectometry (TDR) probe and layered soil system. The model uses a linear-time-invariant feedback system to model each section and links each section in a bottom-up fashion. Multiple sections can be incorporated in this model by a simple extension of a single-section system. An unmatched TDR probe system is modeled by dividing it into equivalent sections and matching the simulated waveform with the actual waveform. The excellent match between the simulated and the recorded waveforms verifies the model. This model eliminates the requirement of a matched 50-Ω probe handler in probe design for dielectric measurement. It can also be used to assist probe design and TDR data interpretation. The reflected waveforms in layered soil specimens are modeled, and the simulated waveforms are compared to those obtained from experiments. The method of using apparent dielectric constant to obtain average volumetric water content is examined for the layered soil system. The results show that apparent dielectric constant is applicable for the measurement of average volumetric water content for a layered soil, but caution has to be used when interpreting the waveforms.

1. Introduction

Over the last 20 years, dramatic developments have been made in the area of time domain reflectometry (TDR). It has become a valuable tool for measuring the moisture content and dielectric properties of soils and other materials. Results of intensive research have been reported on its application in soil moisture measurement [Topp et al., 1980; Roth et al., 1992], development of the TDR probe and the measurement technique [Zegelin et al., 1989; Herkelrath et al., 1991; Heimovaa, 1993], and theoretical modeling [Yanuka et al., 1988; Heimovaa, 1994]. TDR applications cover different fields, including soil science, agriculture, civil engineering, and electrical engineering.

The TDR method consists of two parts: (1) the physical measurement system that includes the TDR apparatus, probe, and other equipment leading to the generation of a consistent TDR waveform and (2) the interpretation of the TDR waveform including its relationship to the desired material property. The success of the interpretation is the key to applying this technology. Topp et al. [1980] established an empirical relationship between the soil volumetric water content and the apparent dielectric constant. This relationship has been widely used and is satisfactory for many applications. Methods have been proposed to measure soil bulk conductivity using TDR [Dalton et al., 1984; Dasberg and Dalton, 1985; Topp et al., 1980], based on the vertical amplitude of the waveform. Recently, a technique to measure soil density has been investigated [Siddiqui and Drnevich; 1995; Feng et al., 1998]. Their study illustrated the significant influence of soil density on the TDR waveform.

This field has also experienced significant theoretical developments. Yanuka et al. [1988] presented a model for multiple reflection and attenuation of TDR pulses. The model is cumulative and can account for many layers of materials. However, the model does not consider the frequency dependency of the material dielectric permittivity. Moreover, the reflections are not complete. Heimovaara [1994] used the S11 scatter function to model the TDR waveform of a matched system. The material dielectric dispersion and loss were modeled in terms of Cole-Cole parameters [Hasted, 1973]. The match of the simulated and the measured waveform was almost exact for some liquids, and the inverse calculation of the dielectric permittivity from the TDR waveform was possible. However, this method is only applicable to a matched system and a homogeneous material.

This paper presents a multisection transmission line model to simulate the TDR waveform from an unmatched probe system and a layered soil system. The model has a general formulation similar to that given by Yanuka et al. [1988], but it uses the phasor form of wave equation and includes the frequency dependency of the material dielectric properties. Results of the simulation are compared to the actual TDR waveforms of some calibration materials, and a simulation of a layered soil system is obtained.
2. Theoretical Background

2.1. Plane Waves in a Dispersive Lossy Material

The electromagnetic properties of a material are characterized by three parameters: (1) dielectric permittivity \( \varepsilon \), (2) electrical conductivity \( \sigma \), and (3) magnetic permeability \( \mu \). In general, these parameters are functions of frequency. However, for materials like soils the magnetic permeability differs from magnetic permeability of free space \( \mu_0 \) by a negligible fraction, and the frequency dependence of conductivity can be neglected. Owing to the dipole polarization mechanism of water, wet soil is a dielectric material that has a frequency-dependent complex relative permittivity:

\[
\varepsilon(f) = \varepsilon_\infty - j\varepsilon'(f) \quad (1)
\]

in which \( f \) is the frequency and \( \varepsilon_\infty \) and \( \varepsilon' \) are the real and imaginary part of permittivity \( \varepsilon \), respectively. The real part of permittivity is often what we call the dielectric constant. It is a measure of how much energy from an external electric field is stored in a material. The imaginary part of permittivity is called the loss factor and is a measure of how dissipative or lossy a material is to an external electric field. The wave equation in phasor form for an electromagnetic (EM) wave propagating in a partially conducting fluid or soil can be established from Maxwell’s equations:

\[
\nabla^2 \hat{E} = \frac{j2\pi f}{\mu_0}(\sigma + j2\pi f\varepsilon)\hat{E} \quad (2a)
\]

\[
\nabla^2 \hat{H} = \frac{j2\pi f}{\mu_0}(\sigma + j2\pi f\varepsilon)\hat{H} \quad (2b)
\]

in which \( \hat{E} \) and \( \hat{H} \) are electric and magnetic fields, respectively, in phasor form; \( j = \sqrt{-1} \). It is convenient to define the equivalent permittivity \( \varepsilon^* \) by equating \( j\omega\varepsilon^* = \sigma + j2\pi f\varepsilon \). In terms of relative permittivity \( (\varepsilon_r = \varepsilon/\varepsilon_0) \), \( \varepsilon^* \) can be derived as

\[
\varepsilon^*_r = \varepsilon_r' - j\left(\varepsilon_r' + \frac{\sigma}{2\pi f\varepsilon_0}\right) \quad (3)
\]

in which \( \varepsilon_0 \) is the dielectric permittivity of free space (equal to \( 8.854 \times 10^{-12} \) F m\(^{-1}\) in MKS system). The imaginary part of equivalent permittivity consists of conductive loss term \( \sigma \) and dielectric loss term \( \varepsilon_r' \) and is defined as

\[
\varepsilon_r'(f) = \varepsilon_\infty'(f) + \frac{\sigma}{2\pi f\varepsilon_0} \quad (4)
\]

A solution to the plane wave equation in (2) at each frequency is

\[
E = \hat{E} e^{j2\pi ft} = E_0 e^{-\alpha z} e^{j2\pi ft - j\beta z} \quad (5a)
\]

\[
H = \hat{H} e^{j2\pi ft} = H_0 e^{-\alpha z} e^{j2\pi ft - j\beta z} \quad (5b)
\]

where \( E \) and \( H \) are electric and magnetic fields, respectively, as a function of time at a certain frequency \( f \); \( E_0 \) and \( H_0 \) are unknown constants in the general solution, which can be determined from the boundary conditions; and \( t \) is the time variable.

The solution to (5) represents a plane wave propagating in the \(+z\) direction of a Cartesian coordinate system. The frequency \( f \), the attenuation constant \( \alpha \), and the phase constant \( \beta \) determine the propagation of the plane wave. The attenuation constant \( \alpha \) and the phase constant \( \beta \) for this material can be expressed as [Ramo et al., 1993]:

\[
\alpha(f) = \frac{2\pi f}{c} \sqrt{\frac{\varepsilon_\infty(f)}{2} \left[ \frac{1 + \varepsilon_r'(f)}{\varepsilon_\infty'(f)} \right]^2 - 1} \quad (6a)
\]

\[
\beta(f) = \frac{2\pi f}{c} \sqrt{\frac{\varepsilon_\infty(f)}{2} \left[ \frac{1 + \varepsilon_r'(f)}{\varepsilon_\infty'(f)} \right]^2 + 1} \quad (6b)
\]

where \( c \) is the speed of light in the free space, \( 2.98 \times 10^8 \) m s\(^{-1}\).

2.2. Soil Dielectric Dispersion Model

Dielectric dispersion and dielectric loss are two characteristic soil dielectric properties. Considering soil as an isotropic and homogeneous media, a Debye type equation modified by Cole and Cole [1941] can be employed to describe the dielectric permittivity:

\[
\varepsilon_s(f) = \varepsilon_\infty' - j\varepsilon_{\infty}'' = \varepsilon_\infty + \frac{\varepsilon_\infty - \varepsilon_\infty'}{1 + (j\omega\tau)^{\alpha}} \quad (7)
\]

where \( \varepsilon_\infty \) and \( \varepsilon_\infty' \) are real values of dielectric permittivity at \( f = 0 \) and \( f \to \infty \), respectively; \( \tau \) is a modified average relaxation frequency related to the molecular polarization; and \( \alpha \) is a parameter representing a distribution of the relaxation time. Substituting (7) into (3), the equivalent permittivity becomes [Heimovaara, 1994]:

\[
\varepsilon^*_s = \varepsilon_\infty + \frac{\varepsilon_\infty - \varepsilon_\infty'}{1 + (j\omega\tau)^{\alpha}} - j\sigma \frac{\alpha}{2\pi f\varepsilon_0} \quad (8)
\]

Some dielectric mixing models have been developed considering that soil is a mixture containing different components [Dobson et al., 1985, Heimovaara et al., 1994]. Those models are either based on De Loor’s [1968] dispersion model or various volumetric mixing models [Birchak et al., 1974]. These models are more sophisticated and can match the testing data well by adjusting the calibration parameters. However, the simpler Cole-Cole functions in (7) or (8) are used in this work.

2.3. Analysis of a Typical TDR Measurement System

A typical TDR measurement system consists of a cable test device, which sends out a step (or other shape) EM wave through a pulse generator and records the incident and reflection signal with an oscilloscope, and a probe that acts as a waveguide to transmit the EM wave into the soil specimen. A step wave or impulse wave is usually used in order to obtain recognizable reflections at the soil surface and the end of the probe.

Transmission line theory is used to model the TDR measurement system in which the field structure in the transmission line is assumed to be of transverse electromagnetic mode (TEM). Owing to the special structure of TEM, (5) can be rewritten in terms of voltage (\( V \)) and current (\( I \)):

\[
V = V_0 e^{-\alpha z} e^{j2\pi ft - j\beta z} \quad (9a)
\]

\[
I = (Z_0 V_0) e^{-\alpha z} e^{j2\pi ft - j\beta z} \quad (9b)
\]

in which \( V \) and \( I \) are voltage and current, respectively; \( V_0 \) is a constant in the general solution which can be determined from the boundary conditions; and \( Z_0 \) is the characteristic impedance. TDR devices typically measure and record the wave in terms of voltage. For a coaxial type probe the characteristic impedance \( Z_0 \) is determined by the radii of the inner and the outer conductor and the dielectric permittivity of the material between them [Heimovaara, 1994].
where $a$ and $b$ are the radii of the inner conductor and the outer conductor of the coaxial container, respectively. When the wave is traveling in a transmission line, attenuation and phase changes occur along the traveling path as presented in (9). The attenuation terms in (9) are collected and defined as the traveling wave function:

$$H(f, z) = \exp \left[ -\alpha(f) z + j\beta(f) z \right] \quad (11)$$

where $\alpha$ is the attenuation constant, $\beta$ is the phase constant defined in (6a) and (6b), and $z$ is the length along the path of propagation.

Characteristic impedance is an intrinsic property of a transmission line. A discontinuity in the characteristic impedance of a transmission line system produces a reflected and a transmitted wave, which can be obtained by multiplication of the incident signal by a reflection coefficient and a transmission coefficient, respectively. The reflection coefficient ($\rho(f)$) and transmission coefficient ($\tau(f)$) in terms of voltage are defined by [Kraus, 1984]

$$\rho(f) = \frac{Z_{c2}(f) - Z_{c1}(f)}{Z_{c2}(f) + Z_{c1}(f)} \quad |\rho(f)| \leq 1 \quad (12)$$

$$\tau(f) = \frac{2Z_{c2}(f)}{Z_{c2}(f) + Z_{c1}(f)} = 1 + \rho(f) \quad |\tau(f)| \leq 1 \quad (13)$$

in which $Z_{c1}$ is the characteristic impedance for the section of the transmission line in which the reflected wave propagates and $Z_{c2}$ is the characteristic impedance for the section of the transmission line in which the transmitted wave propagates. Equations (11)–(13) will serve as the three basic building blocks in this study.

An ideal probe system is matched, which means the probe impedance is the same as that of the connecting cable (50 $\Omega$ for RG174 coaxial cable) up to the soil surface. At the soil surface or the interfaces of an unmatched probe system the wave will be reflected and transmitted. Because the transmission line in a TDR system is of finite length, the wave transmitted into the soil will be reflected at the end of the specimen, which will travel back to the soil surface where it will be reflected and transmitted again. Therefore there are an infinite number of reflections and transmissions that occurs in a TDR system, but the magnitude of the waves keeps decreasing every time they get reflected or transmitted (i.e., $|\rho|, |\tau| \leq 1$). The major focus of TDR waveform simulation is to determine the total reflected wave at the surface of a transmission line from the multiple reflections and transmissions in response to the known incident wave.

The relationship between the total reflected wave ($r(t)$) at the surface of a transmission line and the incident wave ($x(t)$) is a linear-time-invariant (LTI) system and can be described by a scatter function ($S_{11}(f)$) in the frequency domain:

$$R(f) = X(f)S_{11}(f) \quad (14)$$

where $R(f)$ and $X(f)$ are the Fourier transforms of $r(t)$ and $x(t)$, respectively. If $X(f)$ and $S_{11}(f)$ are known, $R(f)$ can be determined using (14), and $r(t)$ is then obtained by inverse Fourier transform of $R(f)$. The TDR device measures the sum of the incident wave and resultant reflected wave (i.e., $x(t) + r(t)$). Section 3 illustrates how to construct the scatter function $S_{11}(f)$ for a multiple-section transmission line from the three basic building blocks (i.e., (11)–(13)).

### 3. Scatter Function for Multisection TDR Measurement System

During field measurement it is possible that within the depth of measurement the soil consists of two or more layers. Also, for some purposes it is much more convenient to use an unmatched probe system. Accordingly, it is desirable to have a complete TDR theoretical model that is capable of simulating both multilayer materials and unmatched probe systems.

The two cases, namely, a layered specimen and an unmatched probe system, actually form the same problem, that is, a multisection transmission system. This is a transmission system consisting of different sections, each with different characteristic impedance. A general description of such a system is shown in Figure 1. The scatter function of a single-section transmission line will be illustrated first. It is then extended to multiple-section transmission lines using a bottom-up approach. Note that the layers of the multiple-section transmission line in Figure 1 are numbered for convenience to illustrate a bottom-up approach.

For a matched, single-layered TDR measurement system the wave propagation scheme and a corresponding system block diagram are given in Figures 2a and 2b. The traveling wave function ($H(f, z)$), reflection coefficient ($\rho(f)$), and transmission coefficient ($\tau(f)$) were given in (11)–(13). The frequency component of the incident signal and the resultant reflected signal are $X(f)e^{j2\pi ft}$ and $R(f)e^{j2\pi ft}$, respectively. A
The scatter function $S_{11}(f)$ can be derived by a simple analysis of the block diagram (see the appendix)

$$S_{11}(f) = \frac{R(f)}{X(f)} = \frac{\rho(f) + H(f, 2L)}{1 + \rho(f)H(f, 2L)}$$

(15)

where $\rho(f)$ is the surface reflection coefficient and $L$ is the length of the probe embedded in the soil. This result is the same as the $S_{11}$ function given by Heimovuura [1994]. The advantage of using this block diagram instead of the $S_{11}(f)$ function is that it allows for separation of the reflected signal $R(f)e^{j2\pi ft}$ from the soil surface and separation of $\sum R_k(f)e^{j2\pi ft}$ from the end of the probe, including the multiple reflections. This gives a clearer description of the traces of wave propagation, comparable to that given by Yanuka et al. [1988].

Now, consider the same system but with the open end reflection coefficient ($\rho_e(f) = 1$) replaced by a specific end reflection coefficient ($\rho_e(f) < 1$). The corresponding system block diagram is shown in Figure 3, and a simple derivation leads to

$$S_{11}(f) = \frac{\rho(f) + \rho_e(f)H(f, 2L)}{1 + \rho(f)\rho_e(f)H(f, 2L)}$$

(16)

where $\rho(f)$ and $\rho_e(f)$ are the surface and end reflection coefficient, respectively. This is a very important building block for the multisection model, which is illustrated below.

The scatter function $S_{11}(f)$ is a system function that derives the total reflection from the input. For a two-section system it is equivalent to replace the first section by an end reflection coefficient $\rho_e(f)$ at the end of the second section that is equal to the scatter function of the removed block, as shown in Figure 4. This is the key concept of this model. The algorithm for the multisection model is developed in a bottom-up style as follows:

1. Model the scatter function of the first section, denoted as $S_{11}^1(f)$, using (16) in which $\rho_s(f)$ is calculated by (12) and $\rho_e(f)$ is determined by a reasonable assumption of the termination, such as an open end (i.e., $\rho_e(f) = 1$).

2. The original $n$-section transmission line can be replaced by an equivalent ($n-1$)-section transmission line which is obtained by removing the original first section and assigning the obtained $S_{11}^1(f)$ to be the end reflection coefficient $\rho_e(f)$ of the equivalent ($n-1$)-section line. Note that the original second section has now become the first section in the equivalent line.

3. Calculate the scatter function for the first section of the equivalent ($n-1$)-section line (originally the second section), denoted as $S_{11}^1(f)$, using (16), in which $\rho_{11}(f)$ is calculated by (12) and $\rho_e(f) = S_{11}^1(f)$ as determined in step 2.

4. Repeat steps 2 and 3 until the last section is reached.

Note that the scatter function for the first section of the equivalent ($n-k$)-section transmission line at the $k$th iteration is denoted as $S_{11}^k(f)$, which actually represents the scatter function for first $k$ sections of the original $n$-section transmission line. The scatter function for the original $n$-section transmission line is $S_{11}^n(f)$ obtained at the $n$th iteration. A block diagram for $S_{11}^n(f)$ is shown in Figure 5. It is interesting to note that the scatter function of the previous section is embedded in the end reflection coefficient $\rho_e(f)$ of the current equivalent ($n-k$)-section line. This makes this algorithm very simple in formulation. Furthermore, every single trace of the wave propagation can be found in the block diagram, assigning a physical meaning to each path in the block diagram. The recursive equation for $S_{11}^n(f)$ is written as

$$S_{11}^n(f) = \frac{\rho_s^k(f) + S_{11}^{n-1}(f)H(f, 2L_k)}{1 + \rho_s^k(f)S_{11}^{n-1}(f)H(f, 2L_k)}$$

(17)

with

$$\rho_s^k(f) = \frac{Z_{k-1}(f) - Z_k(f)}{Z_{k-1}(f) + Z_k(f)}$$

(18)

in which $Z_k$ and $Z_{k-1}$ are the characteristic impedances for $k$th and $(k-1)$th section of the original transmission line, respectively, and $L_k$ is the length of the $k$th section. The scatter
function of the multiple-section transmission line can be obtained after the \( n \)th iteration. The scatter function obtained can then be used to predict the TDR waveforms or inversely determine the dielectric properties from the measured waveforms.

4. Experimental Methods and Materials

4.1. TDR Measurement Device

Measurements were carried out by connecting a coaxial probe to a Tektronix 1502B metallic TDR cable device. The TDR probe system was designed to measure the gravimetric moisture content of compacted soil in a compaction mold. The principle and the procedure of this test were reviewed by Feng et al. [1998]. The TDR probe system consists of a coaxial cable, a coaxial head (CH), and a coaxial cylinder (CC). After the soil is compacted into the coaxial cylinder, a central rod is inserted using a guide plate. The central rod acts as the inner conductor of the coaxial cylinder. The configuration of the coaxial head and the coaxial cylinder are shown in Figures 6a and 6b. The diameter of the central rod and the CC are 7.94 mm (5/16 inch) and 102 mm (4 inches), respectively. The height of the CC mold (or specimen holder) is 203 mm (8 inches). Details of the design information are given by Feng et al. [1998]. The design of the probe system was based on the following considerations: (1) It must be robust to handle high-density compacted soils. (2) The TDR signal must be clear such that the reflections from the surface and the bottom of the soil specimen are recognizable. (3) The electromagnetic connections between different coaxial sections must not have an impact on the quality of the measured signal. Impedance matching from the 50-\( \Omega \) cable to the sensor could not be achieved because of the required geometry.

A detailed simulation of the probe system was needed to fully understand the measurement system and the dielectric properties of the material. The basis for the modeling is that the entire measurement system is assumed to be a multisection transmission line that matches the simulated waveform to the measured waveform. The entire system is composed of four sections: (1) the cable, (2) the coaxial head, (3) the air gap, and (4) the coaxial cylinder. Each section is treated as a coaxial transmission line, with specific geometry (length \( l_i \) and \( b_i/a_i \) ratio) and dielectric constant of the insulating material (modeled by Cole-Cole parameters).

We define two groups of parameters in the modeling, system parameters and material parameters. The system parameters associated with the probe are \( l_i \) and \( b_i/a_i \) of each section and Cole-Cole parameters of the material used in the coaxial head. The material parameters of the soil contained within the CC mold are represented by their Cole-Cole parameters.

4.2. Data Acquisition

When working in the frequency domain, it is important to select an appropriate sample interval and length of the TDR data. The sampling interval is determined by the frequency bandwidth of the TDR measurement system. For the Tektronix 1502B cable tester it was determined to be from dc up to 1.5 GHz [Heimovaara, 1994]. The sample length is selected such that a steady state is reached at the end of the TDR waveform, which means the waveform should achieve a constant value. For a fixed sample interval this appropriate length depends on the dielectric constant of the material and length of the probe.

Computer software, called TDR++, was developed to automate TDR data acquisition and interpretation [Feng et al., 1998]. For this study the program set the 1502B cable tester to a specific horizontal and vertical scale. The horizontal scale is set to 0.1 m per division, and a value of 0.99 is set for the velocity of propagation (\( V_p \)). The sampling frequency under this scale is 37.1 GHz, which is well above twice the TDR frequency bandwidth. The vertical scale was set to 500 \( \text{mrho} \) (where \( \text{mrho} \) is one thousandth of one \( \rho \) and \( \rho \) is the reflection coefficient of a TDR cable system). A 13-bit data quantization scheme is used. The range of data acquisition is from 0 to 8192. The resolution of this scheme is much better than the default 0–128 setting such that we do not need to change the vertical scale to improve the vertical resolution of the waveform. Three data acquisition modes are available for sampling length when
testing different materials, 1024-point, 2048-point, and 4096-point. For most soils, 2048-point waveform is sufficient. However, but for liquids, such as water, a 4096-point waveform is needed to reach a steady state. The sampling time interval between two points is $2.7 \times 10^{-11}$ s, which corresponds to the horizontal scale of 0.1 m per division and $V_p = 0.99$.

Using the horizontal and vertical scale as specified above, the input signal from the Tektronix 1502B is obtained by terminating the cable test device with a 50-Ω impedance block as shown in Figure 7. The sampling frequency under this scale is 37.1 GHz, and the frequency resolution depends on the number of sampling points, with 18.0 MHz for 2048 points and 9 MHz for 4096 points.

4.3. Calibration of the System Parameters

The system parameters are estimated in advance and adjusted to obtain the best possible waveform match. This is necessary because the multisection transmission line model is an idealized model, the air gap section of the CH is not an actual coaxial line, and the dielectric properties of Delrin that are contained in the CH are not well documented. The system parameters are calculated first by using a material with known properties. When the system parameters are obtained such that a good match is achieved, they can be fixed for the TDR and transmission line system including the probe. The only remaining unknowns in the entire system are then the Cole-Cole parameters of the material being tested.

The TDR waveforms for the cable, the cable-coaxial head system, the cable-coaxial head-air gap system and the cable-coaxial head-air gap-empty coaxial cylinder system were recorded. Progressive optimizations were performed to adjust the system parameters, based on the initial values given by the probe geometry and the documented material properties. The optimization was performed in Matlab using the Optimization Toolbox [MathWorks, Inc., 1997]. A least squares error function was defined for each step as

$$\text{erf} = \sum_{i=1}^{n} (M_i - R_i)^2$$  \hspace{1cm} (19)

where $M_i$ are the simulated data, $R_i$ are the recorded data, and $N$ is the total number of the data points. The results were checked by comparing the predicted waveforms to the measured waveforms for both deionized and tap water.

4.4. Simulation and Measurements of a Two-Layered Soil Specimen

Simulation of the TDR waveform for a layered soil specimen has not been reported in the literature. The measured waveforms of layered soil columns were reported by Nadler et al. [1991] for a two-layer system for either wet/dry or dry/wet soil layers. Theoretical modeling and simulation enable a systematic study of the layered system, leading to a deeper understanding of, and a better interpretation of, the TDR waveform.

Using the multisection transmission model presented above, the system can be studied by breaking the specimens into layers. Two layers of soil with two different water contents are compacted into the CC mold and modeled as two sections (dry layer and wet layer), with the total length equal to the length of the CC. The two cases simulated are shown in Figure 8. For the same soil an increase in water content will increase the dielectric permittivity over the whole range of frequency. Reasonable values of Cole-Cole parameters for the dry layer and the wet layer were assumed based on the measurements made by Campbell [1990] for sandy soils. These parameters are listed in Table 1.

In order to verify the simulation results, an experiment was designed and performed using the same probe system. Con-
Table 1. Assumed Cole-Cole Parameters and Conductivity for Dry Specimen and Wet Specimen

<table>
<thead>
<tr>
<th>Cole-Cole Parameters</th>
<th>Dry Sand</th>
<th>Wet Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_s )</td>
<td>3.0</td>
<td>15</td>
</tr>
<tr>
<td>( e_a )</td>
<td>2.8</td>
<td>3</td>
</tr>
<tr>
<td>( f_{\text{rel}, \text{Hz}} )</td>
<td>( 7 \times 10^9 )</td>
<td>( 7 \times 10^9 )</td>
</tr>
<tr>
<td>( \sigma, \text{S m}^{-1} )</td>
<td>0.0002</td>
<td>0.002</td>
</tr>
<tr>
<td>( \alpha' )</td>
<td>0.0125</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

crete sand was used. A wet soil was prepared by adding tap water to obtain gravimetric water content of 14.67%. The air-dried sand was used as the dry soil, with gravimetric water content of 1.75%.

For the dry/wet specimen the wet sand was filled into the CC mold up to about half of the height of the mold. Efforts were taken to make the soil as uniform as possible and make the surface even. The height and weight of the specimen were measured. A thin plastic sheet was used to cover the wet sand and separate it from the dry sand above. Dry sand was then used to fill the CC mold. The total weight of the two-layer specimen was measured. For the wet/dry specimen the same procedure was followed, except that the order of the wet and dry sand was reversed. The volumetric water contents of both the dry and wet parts of each specimen were calculated from the measured gravimetric water content and density, so that the average volumetric water content was known. The results are listed in Table 2. The average volumetric water contents are not the same for the two specimens because it is difficult to prepare specimens with equal thickness and density.

5. Results and Discussion

5.1. Measurement System

A typical TDR waveform of a compacted soil is shown in Figure 9, which is obtained using the program TDR++. The difference between this signal and that obtained from a matched transmission line is the very sharp peak in the waveform. The reflections from the coaxial head and the air gap generate the rising edge before the peak.

The system parameters of the TDR probe are obtained by progressive optimizations, which are performed to minimize the least squares error functions. The initial values and the adjusted system parameters are listed in Table 3. From the results shown in Table 3, it is observed that some of the adjusted parameters differ from the initial values. The idea here is to get “equivalent” system parameters. There are factors in the wave propagation that this TEM behavior may not able to model; for instance, the different geometry of neighboring sections will cause a fringing effect [Wadell, 1991]. However, the purpose of optimization is to get a best matched waveform by adjusting the system parameters.

The optimization is performed section by section since the probe mainly consists of three parts which can be taken apart to make calibration measurements. Trying to consider both geometry and material parameters in the optimization will result in nonuniqueness. Different combinations of \( b/a \) ratio and dielectric constant can result in the same impedance, while different combinations of section length and dielectric constant can result in the same propagation delay. However, it is only required for the coaxial head to have equivalent impedance and propagation delay that will result in correct waveforms.

Most of materials used for the TDR probe head can be assumed lossless and nondispersive (i.e., \( e_s = e_{\infty} \) and \( \sigma = 0 \)). It has not been found that the interfaces cause noticeable loss in signals. Assuming a constant real dielectric constant, the

![Figure 9. A typical time domain reflectometry (TDR) waveform of a compacted soil.](image)

Table 3. Initial Values and Adjusted System Parameters

<table>
<thead>
<tr>
<th>System Parameters</th>
<th>Section 1</th>
<th>Section 2</th>
<th>Section 3</th>
<th>Section 4</th>
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<tbody>
<tr>
<td></td>
<td>Initial Values</td>
<td>Adjusted Values</td>
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<td></td>
</tr>
<tr>
<td>Cable</td>
<td>1.13</td>
<td>1.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( L_c, \text{cm} )</td>
<td>3.33</td>
<td>3.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b/a )</td>
<td>2.1b</td>
<td>2.1b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e_s )</td>
<td>2.1b</td>
<td>2.1b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma, \text{S m}^{-1} )</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha' )</td>
<td>0.0125c</td>
<td>0.0125c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Filling material, Teflon( ^{a} )</td>
<td>6.0</td>
<td>5.98b</td>
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<td></td>
</tr>
<tr>
<td>( L_c, \text{cm} )</td>
<td>9.09</td>
<td>10.0b</td>
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</tr>
<tr>
<td>( b/a )</td>
<td>2.4b</td>
<td>2.4b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f_{\text{rel}, \text{Hz}} )</td>
<td>NK</td>
<td>1.0 \times 10^8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma, \text{S m}^{-1} )</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha' )</td>
<td>0.0125c</td>
<td>0.0125c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Filling material, Delrin( ^{d} )</td>
<td>20.8</td>
<td>20.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( L_c, \text{cm} )</td>
<td>12.8</td>
<td>17.5b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b/a )</td>
<td>12.8</td>
<td>12.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( ^{a} \text{Obtained from Lide [1991].} \)
\( ^{b} \text{Assumed value (solution very insensitive to this value).} \)
\( ^{c} \text{Adjusted values using optimization.} \)
\( ^{d} \text{From Dupont Product Index.} \)
\( ^{e} \text{Filling material, air. For air, } e_s = e_{\infty} = 1.0 \text{ and } \sigma = 0. \)
multisection model can be used to calibrate the system parameters of the commercial TDR probes by dividing the TDR probe into several sections and finding the optimum values of impedance and length for each section.

5.2. Prediction of Waveforms

The results of the system calibration were checked with measurements in materials with known dielectric properties. Deionized water and tap water were used for this purpose. The dielectric properties of those two specimens are listed in Table 4.

The comparisons of the simulated and measured TDR waveform for deionized water and tap water are shown in Figures 10 and 11, respectively. It is observed that the simulated waveforms match the measured ones very well. This justifies the equivalent system parameter concept. A conductivity meter was not available to measure the exact value of the tap water used, so this value was adjusted to best fit the data.

The errors in the early part of the TDR system (i.e., coaxial head) will affect the later part of the waveforms as shown in Figures 10 and 11. These errors may be reduced by a more advanced inverse analysis. However, this error decays with time since the probe head is practically lossless and the waveforms reach the same steady state.

The design of the probe to measure dielectric dispersion is very sophisticated in Heimovaara [1994] in order to achieve a 50-Ω matched probe head. This multisection model eliminates the requirement of a matched system. It allows the flexibility of designing nearly any kind of probe that meets a specific need. The designs should have relatively large transmission coefficients at the interface between the cable and probe head but not necessarily equal to one (i.e., impedance matched). In addition, the multisection model can assist TDR probe design and data interpretation for an application such as water content measurement. Section 5.3 is an example of such application.

5.3. Simulation of Waveforms for Two-Layered Specimens

In the TDR waveform of a uniform soil specimen as shown in Figure 9, the sharp peak is the reflection point due to the soil surface, and the solid vertical line in Figure 9 shows the reflection from the end of the soil specimen. The specific location of the end reflection point can be determined by published methods [Topp et al., 1980; Heimovaara and Bouten 1990]. The end reflection point is somewhat more difficult to determine for layered soil systems as will be discussed below. The apparent length is defined as the distance between these two reflection points, which leads to the apparent dielectric constant $K_a$ [Topp et al., 1980].

The simulation for the dry/wet case and the wet/dry case, described by Figure 8 and Table 1, is shown in Figures 12a. The height of each layer is 102 mm (4 inches), half the length of the CC. It is observed that the TDR waveforms of layered specimens differ markedly compared to those of uniform specimens. For the dry/wet case the waveform drops to a flat portion when hitting the soil surface, then drops rapidly again when hitting the dry/wet interface. The waveform reaches a minimum value before it rises rapidly when hitting the end of the specimen. For the wet/dry case the waveform first drops rapidly when hitting the soil surface and reaches a minimum value at the wet/dry interface, then rises up slowly until having a rapid rising at the end of the specimen. The arrows in Figure 12a illustrate all the reflection points.

From the simulated waveform we can see that special care is needed to identify the reflection point from the end of the specimen. For the dry/wet case the methods for a uniform specimen (tangent line method, etc.) could still be used. However, for the wet/dry case, special caution is needed to differentiate the reflection point because of the wet/dry interface.

### Table 4. Cole-Cole Parameters and Electric Conductivity of Deionized Water and Tap Water

<table>
<thead>
<tr>
<th>Cole-Cole Parameters</th>
<th>Deionized Water</th>
<th>Tap Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$, °C</td>
<td>20.6</td>
<td>20</td>
</tr>
<tr>
<td>$e_r$</td>
<td>79.9</td>
<td>80.2</td>
</tr>
<tr>
<td>$e_i$</td>
<td>4.22</td>
<td>4.22</td>
</tr>
<tr>
<td>$f_{ref}$, GHz</td>
<td>17.0</td>
<td>17.4</td>
</tr>
<tr>
<td>$\sigma$, S m⁻¹</td>
<td>0</td>
<td>0.053b</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0125</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

*According to Heimovaara [1994].
*Adjusted for best fitting.

![Figure 10. Simulated and recorded TDR waveforms for deionized water.](image-url)
from the reflection point at the end of the specimen. The arrival of a wave at the interface of a material with higher impedance causes a point of inflection that is concave upward in the measured waveform. Since the end of the specimen represents an interface of nearly infinite impedance, it represents the last inflection point with upward concavity. However, this final inflection point can be subtle when the dielectric material present at the end of the probe also has very high impedance as with dry soil.

Although only two-layered systems were simulated, a multilayered specimen can be simulated in the same fashion. The simulation provides insight in interpreting the moisture profile from the waveforms. If we have knowledge of a typical waveform for a uniform specimen, a reasonable estimate can be obtained regarding the layered system by comparing the actual waveform to that of a uniform specimen. If the waveform is convex upward first, usually the profile is from dry to wet. If the waveform is concave upward and rises flatly before a rapid rise, it is very possible that the profile is from wet to dry. A more advanced inverse algorithm is being developed to make it possible to determine a moisture profile with depth.

Actual TDR measurements were made for both of the specimens described in Table 2 and Figure 8. The measured waveforms are shown in Figure 12b. It can be observed that the trends of the two waveforms are very similar to the simulated results in Figure 12a. This shows the capability of the multisection model to model layered soil systems. Note that the end reflection points in Figure 12b are not at the same point as shown in Figure 12a because the average water contents are slightly different for the two specimens.

Owing to their simplicity, Cole-Cole parameters were selected to describe the electromagnetic system so that an approach for solving a multilayer TDR system could be illustrated. A more accurate, but more complicated, electromagnetic model for soil could have also been used with this approach such as a volumetric mixing model. The use of a more accurate electromagnetic soil model is likely to produce an even better comparison between the model simulation and the measured electromagnetic response that is illustrated in Figures 12a and 12b.

5.4. Average Apparent Dielectric Constant

The relationship between volumetric moisture content (θ) and the apparent dielectric permittivity (K_a) [Topp et al., 1980] applies to “average” conditions within the soil column. In the simulation of the wet/dry and dry/wet cases the average volumetric water contents are the same; however, an important issue is whether the simulated waveforms give the same apparent dielectric constants. In Figure 12a the reflection points from the surface and the end of the soil specimen are shown. Figure 12a illustrates that the second reflection points are practically the same for the two cases such that accurate interpretation of the waveforms would lead to the same interpreted apparent dielectric constants.

To verify the answer provided by the numerical simulation, the average apparent dielectric constants are measured according to the reading in Figure 12b. The average volumetric moisture contents, calculated by Topp’s equation [Topp et al., 1980], are 12.96 and 14.36% for dry/wet and wet/dry cases, respectively, which are very close to the actual average values (11.86 and 13.14%). This implies that the measurement in the average sense is still applicable for the layered soil, provided that caution is used to read the end reflection in the wet/dry case.

Consequently, it is possible that the TDR method can be used to give the moisture profile in the ground to some depth by taking measurements at progressive depths calculating the averaging volumetric moisture content at each depth. The variation of water content can be obtained in this manner, although the range of depths is limited by the signal energy of the TDR test device.

6. Conclusions

A multisection transmission line model has been presented, and an algorithm to simulate a multisection TDR probe or a layered soil specimen has been implemented. A feedback linear-time-invariant (LTI) system was built for each section, and the scatter function of the previous section is the terminating reflection coefficient of the next section. The model was established in a bottom-up fashion and can account for additional layers. An unmatched TDR probe system was modeled and tested...
using the presented algorithm. System parameters of this probe were determined in the sense of “equivalence” by minimizing the least squares error between predicted waveform and measured waveform. A comparison between the simulated waveforms and the measured ones for some calibration materials demonstrated that this model is valid. The success of the multisection transmission line model eliminates the restriction of probe design for measurement of dielectric dispersion. It can also be used to assist TDR probe design and data interpretation for an application such as water content measurement.

Layered soil specimens were studied using the developed model. The variation in the waveforms for the layered specimens compared to the uniform specimen was examined. Results from the simulation and experiments show that the volumetric moisture content measurement is still applicable to the layered specimen in the average sense, but caution has to be used when interpreting the waveforms. A method to evaluate the moisture profile based on the TDR waveform was proposed.

The purpose of this study is to introduce the feedback control formulation into the multisection TDR systems and to

Figure 12. (a) Simulated waveforms for the dry/wet and wet/dry specimens (b) and recorded waveforms of wet/dry and dry/wet specimens.
show its effectiveness in overcoming the difficulties in modeling multisection dispersive TDR systems. Several inverse problems arise immediately following the success of this forward model, such as whether this model can be used in an inverse analysis that can provide estimates of dielectric variation along the waveguides. An inverse analysis based on this model and probabilistic calculus is under study that will not only allow best estimates of the parameters of interest but will also provide estimates of uncertainty and information on the inverse structure.

Appendix: Derivation of Equation (15)

A complicated block diagram involving feedback loops can be simplified by a step-by-step rearrangement, using rules of block diagram algebra [Ogata, 1997]. Some of these important rules are given in Figure 13. They are obtained by writing the same equation in a different way. The overall transfer function of Figure 2 can be easily derived using these rules of block diagram algebra. Using rule 3 of Figure 13, the feedback loop of Figure 2 can be replaced by an equivalent transfer function:

\[
H/(1 + \rho H)
\]  
(A1)

The overall transfer function can then be calculated using rules 1 and 2:

\[
S_{11}(f) = \rho_s (1 + \rho_s)[H(1 + \rho_s H)](1 - \rho_s)
\]  
(A2)

Simplifying (A2), one obtains (15)

\[
S_{11}(f) = \frac{\rho_s(f) + H(f, 2L)}{1 + \rho_s(f) H(f, 2L)}
\]  
(A3)

Similarly, (16) is derived from Figure 3 using the same rules of block diagram algebra.

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References


